30. Derivatives

Exercise 30.1

1. Question

Find the derivative of f(x) = 3x at x = 2

Answer

Derivative of a function f(x) at any real number a is given by –

$f'(a) = \lim_{h \to 0} \frac{f(a+h)-f(a)}{h}$ {where h is a very small positive number}

∴ derivative of f(x) = 3x at x = 2 is given as –

$$f'(2) = \lim_{h \to 0} \frac{f(2 + h) - f(2)}{h}$$

$$\Rightarrow f'(2) = \lim_{h \to 0} \frac{3(2 + h) - 3 \times 2}{h}$$

$$\Rightarrow f'(2) = \lim_{h \to 0} \frac{3h + 6 - 6}{h} = \lim_{h \to 0} \frac{3h}{h}$$

$$\Rightarrow f'(2) = \lim_{h \to 0} 3 = 3$$

Hence,

Derivative of f(x) = 3x at x = 2 is 3

2. Question

Find the derivative of f(x) = x^2 – 2 at x = 10

Answer

Derivative of a function f(x) at any real number a is given by –

$f'(a) = \lim_{h \to 0} \frac{f(a+h)-f(a)}{h}$ {where h is a very small positive number}

∴ derivative of x^2 – 2 at x = 10 is given as –

$$f'(10) = \lim_{h \to 0} \frac{f(10 + h) - f(10)}{h}$$

$$\Rightarrow f'(10) = \lim_{h \to 0} \frac{(10 + h)^2 - 2 - (10^2 - 2)}{h}$$

$$\Rightarrow f'(10) = \lim_{h \to 0} \frac{100 + h^2 + 20h - 2 - 100 + 2}{h} = \lim_{h \to 0} \frac{h^2 + 20h}{h}$$

$$\Rightarrow f'(10) = \lim_{h \to 0} \frac{h(h + 20)}{h} = \lim_{h \to 0} (h + 20)$$

⇒ f’(10) = 0 + 20 = 20

Hence,

Derivative of f(x) = x^2 – 2 at x = 10 is 20

3. Question

Find the derivative of f(x) = 99x at x = 100.

Answer

Derivative of a function f(x) at any real number a is given by –

$f'(a) = \lim_{h \to 0} \frac{f(a+h)-f(a)}{h}$ {where h is a very small positive number}

∴ derivative of 99x at x = 100 is given as –

$$f'(100) = \lim_{h \to 0} \frac{f(100 + h) - f(100)}{h}$$

$$\Rightarrow f'(100) = \lim_{h \to 0} \frac{99(100 + h) - 99 \times 100}{h}$$

$$\Rightarrow f'(100) = \lim_{h \to 0} \frac{9900 + 99h - 9900}{h} = \lim_{h \to 0} \frac{99h}{h}$$

$$\Rightarrow f'(100) = \lim_{h \to 0} 99 = 99$$

Hence,

Derivative of f(x) = 99x at x = 100 is 99

4. Question

Find the derivative of f(x) = x at x = 1

Answer

Derivative of a function f(x) at any real number a is given by –

$f'(a) = \lim_{h \to 0} \frac{f(a+h)-f(a)}{h}$ {where h is a very small positive number}

∴ derivative of x at x = 1 is given as –

$$f'(1) = \lim_{h \to 0} \frac{f(1 + h) - f(1)}{h}$$

$$\Rightarrow f'(1) = \lim_{h \to 0} \frac{(1 + h) - 1}{h}$$

$$\Rightarrow f'(1) = \lim_{h \to 0} \frac{1 + h - 1}{h} = \lim_{h \to 0} \frac{h}{h}$$

$$\Rightarrow f'(1) = \lim_{h \to 0} 1 = 1$$

Hence,

Derivative of f(x) = x at x = 1 is 1

5. Question

Find the derivative of f(x) = cos x at x = 0

Answer

Derivative of a function f(x) at any real number a is given by –

$f'(a) = \lim_{h \to 0} \frac{f(a+h)-f(a)}{h}$ {where h is a very small positive number}

∴ derivative of cos x at x = 0 is given as –

$$f'(0) = \lim_{h \to 0} \frac{f(0 + h) - f(0)}{h}$$

$$\Rightarrow f'(0) = \lim_{h \to 0} \frac{\cos(h) - \cos 0}{h}$$

$$\Rightarrow f'(0) = \lim_{h \to 0} \frac{\cos h - 1}{h}$$

∵ we can't find the limit by direct substitution as it gives 0/0 (indeterminate form)

So we need to do few simplifications to evaluate the limit.

As we know that 1 – cos x = 2 $\sin^2(x/2)$

$$\therefore f'(0) = \lim_{h \to 0} \frac{-(1 - \cos h)}{h} = -\lim_{h \to 0} \frac{2 \sin^2 \frac{h}{2}}{h}$$

Dividing the numerator and denominator by 2 to get the form (sin x)/x to apply sandwich theorem, also multiplying h in numerator and denominator to get the required form.

$$\Rightarrow f'(0) = -\lim_{h \to 0} \frac{\frac{2 \sin^2 \frac{h}{2}}{2}}{\frac{h^2}{2}} \times h$$

Using algebra of limits we have –

$$\Rightarrow f'(0) = -\lim_{h \to 0} \left(\frac{\sin \frac{h}{2}}{\frac{h}{2}} \right) \times \lim_{h \to 0} h$$

Use the formula: $\lim_{x \to 0} \frac{\sin x}{x} = 1$

∴ f'(0) = – 1×0 = 0

Hence,

Derivative of f(x) = cos x at x = 0 is 0

6. Question

Find the derivative of f(x) = tan x at x = 0

Answer

Derivative of a function f(x) at any real number a is given by –

$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$ {where h is a very small positive number}

∴ derivative of cos x at x = 0 is given as –

$$f'(0) = \lim_{h \to 0} \frac{f(0 + h) - f(0)}{h}$$

$$\Rightarrow f'(0) = \lim_{h \to 0} \frac{\tan(h) - \tan 0}{h}$$

$$\Rightarrow f'(0) = \lim_{h \to 0} \frac{\tan h}{h}$$

∵ we can't find the limit by direct substitution as it gives 0/0 (indeterminate form)

∴ Use the formula: $\lim_{x \to 0} \frac{\tan x}{x} = 1$ {sandwich theorem}

∴ f'(0) = 1

Hence,

Derivative of f(x) = tan x at x = 0 is 1

7 A. Question

Find the derivatives of the following functions at the indicated points :

$\sin x \text{ at } x = \frac{\pi}{2}$

Answer

Derivative of a function f(x) at any real number a is given by –

$f'(a) = \lim_{h \to 0} \frac{f(a+h)-f(a)}{h}$ {where h is a very small positive number}

∴ derivative of sin x at x = π/2 is given as –

$$f'(\pi/2) = \lim_{h \to 0} \frac{f\left(\frac{\pi}{2} + h\right) - f\left(\frac{\pi}{2}\right)}{h}$$

$$\Rightarrow f'(\pi/2) = \lim_{h \to 0} \frac{\sin\left(\frac{\pi}{2} + h\right) - \sin\frac{\pi}{2}}{h}$$

$\Rightarrow f'(\pi/2) = \lim_{h \to 0} \frac{\cos h - 1}{h}$ {∵ sin (π/2 + x) = cos x }

∵ we can't find the limit by direct substitution as it gives 0/0 (indeterminate form)

So we need to do few simplifications to evaluate the limit.

As we know that 1 – cos x = 2 $\sin^2(x/2)$

$$\therefore f'(\pi/2) = \lim_{h \to 0} \frac{-(1-\cos h)}{h} = -\lim_{h \to 0} \frac{2\sin^2\frac{h}{2}}{h}$$

Dividing the numerator and denominator by 2 to get the form (sin x)/x to apply sandwich theorem, also multiplying h in numerator and denominator to get the required form.

$$\Rightarrow f'(\pi/2) = -\lim_{h \to 0} \frac{\frac{2\sin^2\frac{h}{2}}{2}}{\frac{h^2}{2}} \times h$$

Using algebra of limits we have –

$$\Rightarrow f'(\pi/2) = -\lim_{h \to 0} \left(\frac{\sin\frac{h}{2}}{\frac{h}{2}}\right) \times \lim_{h \to 0} h$$

Use the formula: $\lim_{x \to 0} \frac{\sin x}{x} = 1$

∴ f'(π/2) = – 1×0 = 0

Hence,

Derivative of f(x) = sin x at x = π/2 is 0

7 B. Question

Find the derivatives of the following functions at the indicated points :

x at x = 1

Answer

Derivative of a function f(x) at any real number a is given by –

$f'(a) = \lim_{h \to 0} \frac{f(a+h)-f(a)}{h}$ {where h is a very small positive number}

∴ derivative of x at x = 1 is given as –

$$f'(1) = \lim_{h \to 0} \frac{f(1 + h) - f(1)}{h}$$

$$\Rightarrow f'(1) = \lim_{h \to 0} \frac{(1 + h) - 1}{h}$$

$$\Rightarrow f'(1) = \lim_{h \to 0} \frac{1 + h - 1}{h} = \lim_{h \to 0} \frac{h}{h}$$

$$\Rightarrow f'(1) = \lim_{h \to 0} 1 = 1$$

Hence,

Derivative of f(x) = x at x = 1 is 1

7 C. Question

Find the derivatives of the following functions at the indicated points :

$$2\cos x \text{ at } x = \frac{\pi}{2}$$

Answer

Derivative of a function f(x) at any real number a is given by –

$f'(a) = \lim_{h \to 0} \frac{f(a+h)-f(a)}{h}$ {where h is a very small positive number}

∴ derivative of 2cos x at x = π/2 is given as –

$$f'(\pi/2) = \lim_{h \to 0} \frac{f\left(\frac{\pi}{2} + h\right) - f(\frac{\pi}{2})}{h}$$

$$\Rightarrow f'(\pi/2) = \lim_{h \to 0} \frac{2\cos\left(\frac{\pi}{2} + h\right) - 2\cos\frac{\pi}{2}}{h}$$

$\Rightarrow f'(\pi/2) = \lim_{h \to 0} \frac{-2\sin h}{h}$ {∵ cos (π/2 + x) = – sin x }

∵ we can’t find the limit by direct substitution as it gives 0/0 (indeterminate form)

$\Rightarrow f'(\pi/2) = -2 \lim_{h \to 0} \frac{\sin h}{h}$

Use the formula: $\lim_{x \to 0} \frac{\sin x}{x} = 1$

∴ f’(π/2) = – 2×1 = – 2

Hence,

Derivative of f(x) = 2cos x at x = π/2 is – 2

7 D. Question

Find the derivatives of the following functions at the indicated points :

$$\sin 2x \text{ at } x = \frac{\pi}{2}$$

Answer

Derivative of a function f(x) at any real number a is given by –

$f'(a) = \lim_{h \to 0} \frac{f(a+h)-f(a)}{h}$ {where h is a very small positive number}

∴ derivative of sin 2x at x = π/2 is given as –

$$f'(\pi/2) = \lim_{h \to 0} \frac{f\left(\frac{\pi}{2} + h\right) - f\left(\frac{\pi}{2}\right)}{h}$$

$$\Rightarrow f'(\pi/2) = \lim_{h \to 0} \frac{\sin\left\{2 \times \left(\frac{\pi}{2} + h\right)\right\} - \sin 2 \times \frac{\pi}{2}}{h}$$

$$\Rightarrow f'(\pi/2) = \lim_{h \to 0} \frac{\sin(\pi + 2h) - \sin\pi}{h}$$ {∵ sin (π + x) = – sin x & sin π = 0}

$$\Rightarrow f'(\pi/2) = \lim_{h \to 0} \frac{-\sin 2h - 0}{h}$$

$$\Rightarrow f'(\pi/2) = -\lim_{h \to 0} \frac{\sin 2h}{h}$$

∵ we can't find the limit by direct substitution as it gives 0/0 (indeterminate form)

We need to use sandwich theorem to evaluate the limit.

Multiplying 2 in numerator and denominator to apply the formula.

⇒ f'(π/2) = $-\lim_{h \to 0} \frac{\sin 2h}{2h} \times 2 = -2 \lim_{h \to 0} \frac{\sin 2h}{2h}$

Use the formula: $\lim_{x \to 0} \frac{\sin x}{x} = 1$

∴ f'(π/2) = – 2×1 = – 2

Hence,

Derivative of f(x) = sin 2x at x = π/2 is – 2

Exercise 30.2

1 A. Question

Differentiate each of the following from first principles:

$$\frac{2}{x}$$

Answer

We need to find derivative of f(x) = 2/x from first principle

Derivative of a function f(x) from first principle is given by –

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ {where h is a very small positive number}

∴ derivative of f(x) = 2/x is given as –

$$f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}$$

$$\Rightarrow f'(x) = \lim_{h \to 0} \frac{\frac{2}{x + h} - \frac{2}{x}}{h}$$

$$\Rightarrow f'(x) = \lim_{h \to 0} \frac{\frac{2x - 2(x + h)}{x(x + h)}}{h} = \lim_{h \to 0} \frac{2x - 2x - 2h}{h(x)(x + h)}$$

$$\Rightarrow f'(x) = \lim_{h \to 0} \frac{-2h}{h(x)(x + h)} = -2 \lim_{h \to 0} \frac{h}{h(x)(x + h)}$$

As h is cancelled and by putting h = 0 we are not getting any indeterminate form so we can evaluate the limit directly.

$$\therefore f'(x) = -2\frac{1}{x(x+0)} = -\frac{2}{x^2}$$

Hence,

Derivative of f(x) = 2/x = $-\frac{2}{x^2}$

1 B. Question

Differentiate each of the following from first principles:

$$\frac{1}{\sqrt{x}}$$

Answer

We need to find derivative of f(x) = 1/√x

Derivative of a function f(x) from first principle is given by –

$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$ {where h is a very small positive number}

∴ derivative of f(x) = 1/√x is given as –

$$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$$

$$\Rightarrow f'(x) = \lim_{h \to 0} \frac{\frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}}}{h}$$

$$\Rightarrow f'(x) = \lim_{h \to 0} \frac{\frac{\sqrt{x}-\sqrt{x+h}}{\sqrt{x}\sqrt{(x+h)}}}{h} = \lim_{h \to 0} \frac{\sqrt{x}-\sqrt{x+h}}{h\sqrt{x}\sqrt{x+h}}$$

Using algebra of limits –

$$\Rightarrow f'(x) = \lim_{h \to 0} \frac{\sqrt{x}-\sqrt{x+h}}{h} \times \lim_{h \to 0} \frac{1}{\sqrt{x}\sqrt{x+h}}$$

$$\Rightarrow f'(x) = \frac{1}{x} \lim_{h \to 0} \frac{\sqrt{x}-\sqrt{x+h}}{h}$$

Multiplying numerator and denominator by √x + √(x + h) to rationalise the expression so that we don't get any indeterminate form after putting value of h

$$\Rightarrow f'(x) = \frac{1}{x} \lim_{h \to 0} \frac{\sqrt{x}-\sqrt{x+h}}{h} \times \frac{\sqrt{x}+\sqrt{x+h}}{\sqrt{x}+\sqrt{x+h}}$$

Using (a + b)(a – b) = a^2 – b^2

$$f'(x) = \frac{1}{x} \lim_{h \to 0} \frac{(\sqrt{x})^2 - (\sqrt{x+h})^2}{h} \times \frac{1}{\sqrt{x}+\sqrt{x+h}}$$

Using algebra of limits –

$$f'(x) = \frac{1}{x} \lim_{h \to 0} \frac{x-x-h}{h} \times \lim_{h \to 0} \frac{1}{\sqrt{x}+\sqrt{x+h}}$$

$$\Rightarrow f'(x) = \frac{1}{x} \times (-1) \times \frac{1}{2\sqrt{x}}$$

$\therefore\ f'(x) = -1 \times \frac{1}{2x\sqrt{x}} = -\frac{1}{2x\sqrt{x}}$

Hence,

Derivative of f(x) = 1/√x = $-\frac{1}{2x\sqrt{x}}$

1 C. Question

Differentiate each of the following from first principles:

$$\frac{1}{x^3}$$

Answer

We need to find derivative of f(x) = $1/x^3$

Derivative of a function f(x) from first principle is given by –

$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$ {where h is a very small positive number}

∴ derivative of f(x) = $1/x^3$ is given as –

$$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$$

$$\Rightarrow f'(x) = \lim_{h \to 0} \frac{\frac{1}{(x+h)^3} - \frac{1}{x^3}}{h}$$

$$\Rightarrow f'(x) = \lim_{h \to 0} \frac{\frac{x^3-(x+h)^3}{x^3(x+h)^3}}{h} = \lim_{h \to 0} \frac{x^3-(x+h)^3}{h(x^3)(x+h)^3}$$

Using algebra of limits –

$$\Rightarrow f'(x) = \lim_{h \to 0} \frac{x^3-(x+h)^3}{h} \times \lim_{h \to 0} \frac{1}{(x)^3(x+h)^3}$$

$$\Rightarrow f'(x) = \frac{1}{x^6} \lim_{h \to 0} \frac{x^3-(x+h)^3}{h}$$

Using $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

We have:

$$f'(x) = \frac{1}{x^6} \lim_{h \to 0} \frac{(x-x-h)(x^2 + x(x+h) + (x+h)^2)}{h}$$

$$\Rightarrow f'(x) = \frac{1}{x^6} \lim_{h \to 0} \frac{-h(x^2 + x(x+h) + h^2 + x^2 + 2xh)}{h}$$

As h is cancelled and by putting h = 0 we are not getting any indeterminate form so we can evaluate the limit directly.

$$\Rightarrow f'(x) = -\frac{1}{x^6} \lim_{h \to 0} (x^2 + x^2 + xh + x^2 + 2xh + h^2)$$

$$\Rightarrow f'(x) = -\frac{1}{x^6} \lim_{h \to 0} (3x^2 + h^2 + 3xh)$$

$$\Rightarrow f'(x) = -\frac{1}{x^6} (3x^2 + 3x(0) + 0^2) = -\frac{3x^2}{x^6} = -\frac{3}{x^4}$$

$\therefore f'(x) = -\frac{3}{x^4}$

Hence,

Derivative of f(x) = $1/x^3 = -\frac{3}{x^4}$

1 D. Question

Differentiate each of the following from first principles:

$$\frac{x^2+1}{x}$$

Answer

We need to find derivative of $f(x) = \frac{x^2+1}{x}$

Derivative of a function f(x) from first principle is given by –

$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$ {where h is a very small positive number}

$\therefore$ derivative of $f(x) = \frac{x^2+1}{x}$ is given as –

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$\Rightarrow f'(x) = \lim_{h\to 0} \frac{\frac{(x+h)^2+1}{x+h} - \frac{x^2+1}{x}}{h}$$

$$\Rightarrow f'(x) = \lim_{h\to 0} \frac{\frac{\{(x+h)^2+1\}x - (x+h)(x^2+1)}{x(x+h)}}{h}$$

$$= \lim_{h\to 0} \frac{\{(x+h)^2+1\}x - (x+h)(x^2+1)}{hx(x+h)}$$

Using algebra of limits –

$$\Rightarrow f'(x) = \lim_{h\to 0} \frac{\{(x+h)^2+1\}x - (x+h)(x^2+1)}{h} \times \lim_{h\to 0} \frac{1}{x(x+h)}$$

$$\Rightarrow f'(x) = \frac{1}{x^2} \lim_{h\to 0} \frac{\{(x+h)^2+1\}x - (x+h)(x^2+1)}{h}$$

$$\Rightarrow f'(x) = \frac{1}{x^2} \lim_{h\to 0} \frac{\{x^2+h^2+2xh+1\}x - \{x^3+hx^2+x+h\}}{h}$$

$$\Rightarrow f'(x) = \frac{1}{x^2} \lim_{h\to 0} \frac{x^3+h^2x+2x^2h+x-x^3-hx^2-x-h}{h}$$

$$\Rightarrow f'(x) = \frac{1}{x^2} \lim_{h\to 0} \frac{h^2x+x^2h-h}{h}$$

$$\Rightarrow f'(x) = \frac{1}{x^2} \lim_{h\to 0} \frac{h(hx+x^2-1)}{h}$$

As h is cancelled and by putting h = 0 we are not getting any indeterminate form so we can evaluate the limit directly.

$$\Rightarrow f'(x) = \frac{1}{x^2} \lim_{h\to 0} (hx+x^2-1)$$

$$\Rightarrow f'(x) = \frac{1}{x^2}(0 \times x + x^2 - 1) = \frac{x^2-1}{x^2} = 1 - \frac{1}{x^2}$$

$$\therefore f'(x) = 1 - \frac{1}{x^2}$$

Hence,

Derivative of f(x) = $\frac{x^2+1}{x} = 1 - \frac{1}{x^2}$

1 E. Question

Differentiate each of the following from first principles:

$$\frac{x^2-1}{x}$$

Answer

We need to find derivative of f(x) = $\frac{x^2-1}{x}$

Derivative of a function f(x) from first principle is given by –

$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$ {where h is a very small positive number}

∴ derivative of f(x) = $\frac{x^2-1}{x}$ is given as –

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\Rightarrow f'(x) = \lim_{h \to 0} \frac{\frac{(x+h)^2-1}{x+h} - \frac{x^2-1}{x}}{h}$$

$$\Rightarrow f'(x) = \lim_{h \to 0} \frac{\frac{\{(x+h)^2-1\}x - (x+h)(x^2-1)}{x(x+h)}}{h}$$

$$= \lim_{h \to 0} \frac{\{(x+h)^2-1\}x - (x+h)(x^2-1)}{hx(x+h)}$$

Using algebra of limits –

$$\Rightarrow f'(x) = \lim_{h \to 0} \frac{\{(x+h)^2-1\}x - (x+h)(x^2-1)}{h} \times \lim_{h \to 0} \frac{1}{x(x+h)}$$

$$\Rightarrow f'(x) = \frac{1}{x^2} \lim_{h \to 0} \frac{\{(x+h)^2-1\}x - (x+h)(x^2-1)}{h}$$

$$\Rightarrow f'(x) = \frac{1}{x^2} \lim_{h \to 0} \frac{\{x^2 + h^2 + 2xh - 1\}x - \{x^3 + hx^2 - x - h\}}{h}$$

$$\Rightarrow f'(x) = \frac{1}{x^2} \lim_{h \to 0} \frac{x^3 + h^2x + 2x^2h - x - x^3 - hx^2 + x + h}{h}$$

$$\Rightarrow f'(x) = \frac{1}{x^2} \lim_{h \to 0} \frac{h^2x + x^2h + h}{h}$$

$$\Rightarrow f'(x) = \frac{1}{x^2} \lim_{h \to 0} \frac{h(hx + x^2 + 1)}{h}$$

As h is cancelled and by putting h = 0 we are not getting any indeterminate form so we can evaluate the limit directly.

$$\Rightarrow f'(x) = \frac{1}{x^2}\lim_{h\to 0}(hx + x^2 + 1)$$

$$\Rightarrow f'(x) = \frac{1}{x^2}(0 \times x + x^2 + 1) = \frac{x^2 + 1}{x^2} = 1 + \frac{1}{x^2}$$

$$\therefore f'(x) = 1 + \frac{1}{x^2}$$

Hence,

Derivative of f(x) $= \frac{x^2+1}{x} = 1 + \frac{1}{x^2}$

1 F. Question

Differentiate each of the following from first principles:

$$\frac{x+1}{x+2}$$

Answer

We need to find derivative of $f(x) = \frac{x+1}{x+2}$

Derivative of a function f(x) from first principle is given by –

$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ {where h is a very small positive number}

∴ derivative of $f(x) = \frac{x+1}{x+2}$ is given as –

$$f'(x) = \lim_{h\to 0}\frac{f(x + h) - f(x)}{h}$$

$$\Rightarrow f'(x) = \lim_{h\to 0}\frac{\frac{(x + h) + 1}{x + h + 2} - \frac{x + 1}{x + 2}}{h}$$

$$\Rightarrow f'(x) = \lim_{h\to 0}\frac{\frac{\{x + h + 1\}\{x + 2\} - (x + h + 2)(x + 1)}{(x + 2)(x + h + 2)}}{h}$$
$$= \lim_{h\to 0}\frac{\{x + h + 1\}\{x + 2\} - (x + h + 2)(x + 1))}{h(x + 2)(x + h + 2)}$$

Using algebra of limits –

$$\Rightarrow f'(x) = \lim_{h\to 0}\frac{\{x + h + 1\}\{x + 2\} - (x + h + 2)(x + 1)}{h} \times \lim_{h\to 0}\frac{1}{(x + 2)(x + h + 2)}$$

$$\Rightarrow f'(x) = \frac{1}{(x + 2)^2}\lim_{h\to 0}\frac{\{x + h + 1\}\{x + 2\} - (x + h + 2)(x + 1)}{h}$$

$$\Rightarrow f'(x) = \frac{1}{(x + 2)^2}\lim_{h\to 0}\frac{x^2 + 2x + hx + 2h + x + 2 - x^2 - x - hx - h - 2x - 2}{h}$$

$$\Rightarrow f'(x) = \frac{1}{(x + 2)^2}\lim_{h\to 0}\frac{h}{h}$$

$$\Rightarrow f'(x) = \frac{1}{(x + 2)^2}\lim_{h\to 0}1$$

$\therefore\ f'(x) = \frac{1}{(x+2)^2}$

Hence,

Derivative of f(x) = $\frac{x+1}{x+2} = \frac{1}{(x+2)^2}$

1 G. Question

Differentiate each of the following from first principles:

$$\frac{x+2}{3x+5}$$

Answer

We need to find derivative of $f(x) = \frac{x+2}{3x+5}$

Derivative of a function f(x) is given by –

$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$ {where h is a very small positive number}

$\therefore$ derivative of $f(x) = \frac{x+2}{3x+5}$ is given as –

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\Rightarrow f'(x) = \lim_{h \to 0} \frac{\frac{(x+h)+2}{3(x+h)+5} - \frac{x+2}{3x+5}}{h}$$

$$\Rightarrow f'(x) = \lim_{h \to 0} \frac{\frac{\{x+h+2\}\{3x+5\} - (3x+3h+5)(x+2)}{(3x+5)(3x+3h+5)}}{h}$$

$$\Rightarrow f'(x) = \lim_{h \to 0} \frac{\{x+h+2\}\{3x+5\} - (3x+3h+5)(x+2)}{h(3x+5)(3x+3h+5)}$$

Using algebra of limits –

$$\Rightarrow f'(x) = \lim_{h \to 0} \frac{\{x+h+2\}\{3x+5\} - (3x+3h+5)(x+2)}{h} \times \lim_{h \to 0} \frac{1}{(3x+5)(3x+3h+5)}$$

$$\Rightarrow f'(x) = \frac{1}{(3x+5)^2} \lim_{h \to 0} \frac{\{x+h+2\}\{3x+5\} - (3x+3h+5)(x+2)}{h}$$

$$\Rightarrow f'(x) = \frac{1}{(3x+5)^2} \lim_{h \to 0} \frac{3x^2 + 5x + 3hx + 5h + 6x + 10 - 3x^2 - 6x - 3hx - 6h - 5x - 10}{h}$$

$$\Rightarrow f'(x) = \frac{1}{(3x+5)^2} \lim_{h \to 0} \frac{5h - 6h}{h}$$

$$\Rightarrow f'(x) = \frac{1}{(3x+5)^2} \lim_{h \to 0} \frac{-h}{h}$$

$$\Rightarrow f'(x) = \frac{1}{(3x+5)^2} \lim_{h \to 0} -1$$

$$\therefore f'(x) = \frac{-1}{(3x+5)^2}$$

Hence,

Derivative of f(x) = $\frac{x+2}{3x+5} = \frac{-1}{(3x+5)^2}$

1 H. Question

Differentiate each of the following from first principles:

kx^n

Answer

We need to find the derivative of $f(x) = kx^n$

Derivative of a function f(x) from first principle is given by –

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ {where h is a very small positive number}

∴ derivative of $f(x) = kx^n$ is given as –

$$f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}$$

$$\Rightarrow f'(x) = \lim_{h \to 0} \frac{k(x + h)^n - kx^n}{h}$$

$$\Rightarrow f'(x) = k \lim_{h \to 0} \frac{(x + h)^n - x^n}{h}$$

Using binomial expansion we have –

$(x + h)^n = {}^nC_0\, x^n + {}^nC_1\, x^{n-1}h + {}^nC_2\, x^{n-2}h^2 + \ldots\ldots + {}^nC_n\, h^n$

$$\therefore f'(x) = k \lim_{h \to 0} \frac{x^n + {}^nC_1 x^{n-1}h + {}^nC_2 x^{n-2} + \ldots + {}^nC_n h^n - x^n}{h}$$

$$\Rightarrow f'(x) = k \lim_{h \to 0} \frac{{}^nC_1 x^{n-1}h + {}^nC_2 x^{n-2}h^2 + \ldots + {}^nC_n h^n}{h}$$

Take h common –

$$\Rightarrow f'(x) = k \lim_{h \to 0} \frac{h(+{}^nC_1 x^{n-1} + {}^nC_2 x^{n-2}h + \ldots + {}^nC_n h^{n-1})}{h}$$

$$\Rightarrow f'(x) = k \lim_{h \to 0} (+{}^nC_1 x^{n-1} + {}^nC_2 x^{n-2}h + \ldots + {}^nC_n h^{n-1})$$

As there is no more indeterminate, so put value of h to get the limit.

$$\Rightarrow f'(x) = k \lim_{h \to 0} (+{}^nC_1 x^{n-1} + {}^nC_2 x^{n-2}0 + \ldots + {}^nC_n 0^{n-1})$$

$\Rightarrow f'(x) = k\, {}^nC_1\, x^{n-1} = k\, nx^{n-1}$

Hence,

Derivative of $f(x) = kx^n$ is $k\, nx^{n-1}$

1 I. Question

Differentiate each of the following from first principles:

$$\frac{1}{\sqrt{3-x}}$$

Answer

We need to find derivative of $f(x) = 1/\sqrt{(3 - x)}$

Derivative of a function f(x) from first principle is given by –

$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$ {where h is a very small positive number}

∴ derivative of f(x) = 1/√(3 – x) is given as –

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\Rightarrow f'(x) = \lim_{h \to 0} \frac{\frac{1}{\sqrt{3-(x+h)}} - \frac{1}{\sqrt{3-x}}}{h}$$

$$\Rightarrow f'(x) = \lim_{h \to 0} \frac{\frac{\sqrt{3-x} - \sqrt{3-x-h}}{\sqrt{3-x}\sqrt{(3-x-h)}}}{h} = \lim_{h \to 0} \frac{\sqrt{3-x} - \sqrt{3-x-h}}{h\sqrt{3-x}\sqrt{3-x-h}}$$

Using algebra of limits –

$$\Rightarrow f'(x) = \lim_{h \to 0} \frac{\sqrt{3-x} - \sqrt{3-x-h}}{h} \times \lim_{h \to 0} \frac{1}{\sqrt{3-x}\sqrt{3-x-h}}$$

$$\Rightarrow f'(x) = \frac{1}{(3-x)} \lim_{h \to 0} \frac{\sqrt{3-x} - \sqrt{3-x-h}}{h}$$

Multiplying numerator and denominator by √(3 – x) + √(3 – x – h) to rationalise the expression so that we don't get any indeterminate form after putting value of h

$$\Rightarrow f'(x) = \frac{1}{(3-x)} \lim_{h \to 0} \frac{\sqrt{3-x} - \sqrt{3-x-h}}{h} \times \frac{\sqrt{3-x} + \sqrt{3-x-h}}{\sqrt{3-x} + \sqrt{3-x-h}}$$

Using (a + b)(a – b) = $a^2 - b^2$

$$f'(x) = \frac{1}{3-x} \lim_{h \to 0} \frac{\left(\sqrt{3-x}\right)^2 - \left(\sqrt{3-x-h}\right)^2}{h} \times \frac{1}{\sqrt{3-x} + \sqrt{3-x-h}}$$

Using algebra of limits –

$$f'(x) = \frac{1}{3-x} \lim_{h \to 0} \frac{3-x-(3-x-h)}{h} \times \lim_{h \to 0} \frac{1}{\sqrt{3-x} + \sqrt{3-x-h}}$$

$$\Rightarrow f'(x) = \frac{1}{(3-x)} \times (1) \times \frac{1}{2\sqrt{3-x}}$$

$$\therefore f'(x) = 1 \times \frac{1}{2(3-x)\sqrt{3-x}} = \frac{1}{2(3-x)\sqrt{3-x}}$$

Hence,

$$\text{Derivative of } \left(f(x) = \frac{1}{\sqrt{x}}\right) = \frac{1}{2(3-x)\sqrt{3-x}}$$

1 J. Question

Differentiate each of the following from first principles:

$x^2 + x + 3$

Answer

We need to find the derivative of f(x) = $x^2 + x + 3$

Derivative of a function f(x) from first principle is given by –

$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$ {where h is a very small positive number}

∴ derivative of f(x) = $x^2 + x + 3$ is given as –

$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$

$\Rightarrow f'(x) = \lim_{h \to 0} \frac{(x+h)^2 + (x+h) + 3 - (x^2 + x + 3)}{h}$

Using $(a + b)^2 = a^2 + 2ab + b^2$

$\Rightarrow f'(x) = \lim_{h \to 0} \frac{x^2 + 2xh + h^2 + (x+h) + 3 - (x^2 + x + 3)}{h}$

$\Rightarrow f'(x) = \lim_{h \to 0} \frac{2xh + h^2 + h}{h}$

Take h common –

$\Rightarrow f'(x) = \lim_{h \to 0} \frac{h(2x + 1 + h)}{h}$

$\Rightarrow f'(x) = \lim_{h \to 0} (2x + h + 1)$

As there is no more indeterminate, so put value of h to get the limit.

$\Rightarrow f'(x) = (2x + 0 + 1)$

$\Rightarrow f'(x) = 2x + 1 = 2x + 1$

Hence,

Derivative of f(x) = $x^2 + x + 3$ is $(2x + 1)$

1 K. Question

Differentiate each of the following from first principles:

$(x + 2)^3$

Answer

We need to find the derivative of f(x) = $(x + 2)^3$

Derivative of a function f(x) from first principle is given by –

$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$ {where h is a very small positive number}

∴ derivative of f(x) = $(x + 2)^3$ is given as –

$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$

$\Rightarrow f'(x) = \lim_{h \to 0} \frac{(x+2+h)^3 - (x+2)^3}{h}$

Using $a^3 – b^3 = (a – b)(a^2 + ab + b^2)$

$\Rightarrow f'(x) = \lim_{h \to 0} \frac{(x+h+2-x-2)((x+h+2)^2 + (x+h+2)(x+2) + (x+2)^2)}{h}$

$\Rightarrow f'(x) = \lim_{h \to 0} \frac{(h)\{(x+h+2)^2 + (x+h+2)(x+2) + (x+2)^2\}}{h}$

As h is cancelled, so there is no more indeterminate form possible if we put value of h = 0.

So, evaluate the limit by putting h = 0

$\Rightarrow f'(x) = \lim_{h \to 0} \{(x + h + 2)^2 + (x + h + 2)(x + 2) + (x + 2)^2\}$

$\Rightarrow f'(x) = (x + 0 + 2)^2 + (x + 2)(x + 2) + (x + 2)^2$

$\Rightarrow f'(x) = 3\ (x + 2)^2$

$\Rightarrow f'(x) = 3\ (x + 2)^2$

Hence,

Derivative of $f(x) = (x + 2)^3$ is $3(x + 2)^2$

1 L. Question

Differentiate each of the following from first principles:

$x^3 + 4x^2 + 3x + 2$

Answer

We need to find the derivative of $f(x) = x^3 + 4x^2 + 3x + 2$

Derivative of a function f(x) from first principle is given by –

$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$ {where h is a very small positive number}

∴ derivative of $f(x) = x^3 + 4x^2 + 3x + 2$is given as –

$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$

$\Rightarrow f'(x) = \lim_{h \to 0} \frac{(x+h)^3 + 4(x+h)^2 + 3(x+h) + 2 - (x^3 + 4x^2 + 3x + 2)}{h}$

$\Rightarrow f'(x) = \lim_{h \to 0} \frac{(x+h)\{(x+h)^2 + 4(x+h) + 3\} + 2 - x^3 - 4x^2 - 3x - 2}{h}$

Using $(a + b)^2 = a^2 + 2ab + b^2$, we have –

$\Rightarrow f'(x) = \lim_{h \to 0} \frac{(x+h)\{x^2 + 2xh + h^2 + 4x + 4h + 3\} - x^3 - 4x^2 - 3x}{h}$

$\Rightarrow f'(x) = \lim_{h \to 0} \frac{x^3 + 3x^2h + 3xh^2 + 4x^2 + 8hx + 3x + 3h + h^3 + 4h^2 - x^3 - 4x^2 - 3x}{h}$

$\Rightarrow f'(x) = \lim_{h \to 0} \frac{3x^2h + 3xh^2 + 8hx + 3h + h^3 + 4h^2}{h}$

Take h common –

$\Rightarrow f'(x) = \lim_{h \to 0} \frac{h(3x^2 + 3xh + 8x + 3 + h^2 + 4h)}{h}$

As h is cancelled, so there is no more indeterminate form possible if we put value of h = 0.

So, evaluate the limit by putting h = 0

$\Rightarrow f'(x) = \lim_{h \to 0} (3x^2 + 3xh + 8x + 3 + h^2 + 4h)$

$\Rightarrow f'(x) = 3x^2 + 3x(0) + 8x + 3 + 0^2 + 4(0)$

$\Rightarrow f'(x) = 3x^2 + 8x + 3$

Hence,

Derivative of $f(x) = x^3 + 4x^2 + 3x + 2$ is $3x^2 + 8x + 3$

1 M. Question

Differentiate each of the following from first principles:

$(x^2 + 1)(x - 5)$

Answer

We need to find the derivative of $f(x) = (x^2 + 1)(x - 5)$

Derivative of a function f(x) from first principle is given by –

$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$ {where h is a very small positive number}

∴ derivative of $f(x) = (x^2 + 1)(x - 5)$ is given as –

$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$

$\Rightarrow f'(x) = \lim_{h \to 0} \frac{\{(x+h)^2+1\}\{x+h-5\}-(x^2+1)(x-5)}{h}$

$\Rightarrow f'(x) = \lim_{h \to 0} \frac{\{(x+h)^3+x+h-5(x+h)^2-5\}-(x^3-5x^2+x-5)}{h}$

Using $(a + b)^2 = a^2 + 2ab + b^2$ and $(a + b)^3 = a^3 + 3ab(a + b) + b^3$ we have –

$\Rightarrow f'(x) = \lim_{h \to 0} \frac{\{x^3+3x^2h+3h^2x+h^3+x+h-5x^2-10hx-5h^2-5\}-(x^3-5x^2+x-5)}{h}$

$\Rightarrow f'(x) = \lim_{h \to 0} \frac{\{3x^2h+3h^2x+h^3+h-10hx-5h^2\}}{h}$

Take h common –

$\Rightarrow f'(x) = \lim_{h \to 0} \frac{h\{3x^2+3hx+h^2+1-10x-5h\}}{h}$

As h is cancelled, so there is no more indeterminate form possible if we put value of h = 0

$\therefore f'(x) = \lim_{h \to 0} \{3x^2 + 3hx + h^2 + 1 - 10x - 5h\}$

So, evaluate the limit by putting h = 0

$\Rightarrow f'(x) = 3x^2 + 3(0)x + 0^2 + 1 - 10x - 5(0)$

$\Rightarrow f'(x) = 3x^2 - 10x + 1$

Hence,

Derivative of $f(x) = (x^2 + 1)(x - 5)$ is $3x^2 - 10x + 1$

1 N. Question

Differentiate each of the following from first principles:

$\sqrt{2x^2+1}$

Answer

We need to find derivative of $f(x) = \sqrt{(2x^2 + 1)}$

Derivative of a function f(x) from first principle is given by –

$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$ {where h is a very small positive number}

∴ derivative of $f(x) = \sqrt{(2x^2 + 1)}$ is given as –

$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$

$\Rightarrow f'(x) = \lim_{h \to 0} \frac{\sqrt{2(x+h)^2+1}-\sqrt{2x^2+1}}{h}$

As the above limit can't be evaluated by putting the value of h because it takes 0/0 (indeterminate form)

∴ multiplying denominator and numerator by $\sqrt{2(x + h)^2 + 1} + \sqrt{2x^2 + 1}$ to eliminate the indeterminate form.

$\Rightarrow f'(x) = \lim_{h \to 0} \frac{\sqrt{2(x+h)^2+1}-\sqrt{2x^2+1}}{h} \times \frac{\sqrt{2(x+h)^2+1}+\sqrt{2x^2+1}}{\sqrt{2(x+h)^2+1}+\sqrt{2x^2+1}}$

Using algebra of limits & $a^2 - b^2 = (a + b)(a - b)$,we have –

$\Rightarrow f'(x) = \lim_{h \to 0} \frac{\left(\sqrt{2(x+h)^2+1}\right)^2-\left(\sqrt{2x^2+1}\right)^2}{h} \times \lim_{h \to 0} \frac{1}{\sqrt{2(x+h)^2+1}+\sqrt{2x^2+1}}$

$\Rightarrow f'(x) = \frac{1}{2\sqrt{2x^2+1}} \lim_{h \to 0} \frac{2(x+h)^2+1-2x^2-1}{h}$

$\Rightarrow f'(x) = \frac{2}{2\sqrt{2x^2+1}} \lim_{h \to 0} \frac{(x+h)^2-x^2}{h}$

Using $a^2 - b^2 = (a + b)(a - b)$, we have –

$\Rightarrow f'(x) = \frac{2}{2\sqrt{2x^2+1}} \lim_{h \to 0} \frac{(x+h-x)(x+h+x)}{h}$

$\Rightarrow f'(x) = \frac{1}{\sqrt{2x^2+1}} \lim_{h \to 0} \frac{h(2x+h)}{h}$

$\Rightarrow f'(x) = \frac{1}{\sqrt{2x^2+1}} \lim_{h \to 0} (2x + h)$

Evaluating the limit by putting h = 0

$\therefore f'(x) = \frac{1}{\sqrt{2x^2+1}} (2x + 0)$

$\therefore f'(x) = \frac{2x}{\sqrt{2x^2+1}}$

Hence,

Derivative of $f(x) = \sqrt{(2x^2 + 1)} = \frac{2x}{\sqrt{2x^2+1}}$

1 O. Question

Differentiate each of the following from first principles:

$$\frac{2x+3}{x-2}$$

Answer

We need to find derivative of $f(x) = \frac{2x+3}{x-2}$

Derivative of a function f(x) from first principle is given by –

$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$ {where h is a very small positive number}

∴ derivative of $f(x) = \frac{2x+3}{x-2}$ is given as –

$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$

$\Rightarrow f'(x) = \lim_{h \to 0} \frac{\frac{2(x+h)+3}{x+h-2}-\frac{2x+3}{x-2}}{h}$

$\Rightarrow f'(x) = \lim_{h \to 0} \frac{\frac{\{2x+2h+3\}\{x-2\}-(x+h-2)(2x+3)}{(x-2)(x+h-2)}}{h}$

$\Rightarrow f'(x) = \lim\limits_{h \to 0} \frac{\{2x+2h+3\}\{x-2\}-(x+h-2)(2x+3)}{h(x-2)(x+h-2)}$

Using algebra of limits –

$\Rightarrow f'(x) = \lim\limits_{h \to 0} \frac{\{2x+2h+3\}\{x-2\}-(x+h-2)(2x+3)}{h} \times \lim\limits_{h \to 0} \frac{1}{(x-2)(x+h-2)}$

$\Rightarrow f'(x) = \frac{1}{(x-2)^2} \lim\limits_{h \to 0} \frac{\{2x+2h+3\}\{x-2\}-(x+h-2)(2x+3)}{h}$

$\Rightarrow f'(x) = \frac{1}{(x-2)^2} \lim\limits_{h \to 0} \frac{2x^2-4x+2hx-4h+3x-6-2x^2-3x-2hx-3h+4x+6}{h}$

$\Rightarrow f'(x) = \frac{1}{(x-2)^2} \lim\limits_{h \to 0} \frac{-7h}{h}$

$\Rightarrow f'(x) = \frac{1}{(x-2)^2} \lim\limits_{h \to 0} -7$

$\therefore f'(x) = -\frac{7}{(x-2)^2}$

Hence,

Derivative of f(x) $= \frac{2x+3}{x-2} = -\frac{7}{(x-2)^2}$

2 A. Question

Differentiate the following from first principle.

e^{-x}

Answer

We need to find derivative of $f(x) = e^{-x}$

Derivative of a function f(x) is given by –

$f'(x) = \lim\limits_{h \to 0} \frac{f(x+h)-f(x)}{h}$ {where h is a very small positive number}

$\therefore$ derivative of $f(x) = e^{-x}$ is given as –

$f'(x) = \lim\limits_{h \to 0} \frac{f(x+h)-f(x)}{h}$

$\Rightarrow f'(x) = \lim\limits_{h \to 0} \frac{e^{-(x+h)}-e^{-x}}{h}$

$\Rightarrow f'(x) = \lim\limits_{h \to 0} \frac{e^{-x}e^{-h}-e^{-x}}{h}$

Taking e^{-x} common, we have –

$\Rightarrow f'(x) = \lim\limits_{h \to 0} \frac{e^{-x}(e^{-h}-1)}{h}$

Using algebra of limits –

$\Rightarrow f'(x) = \lim\limits_{h \to 0} e^{-x} \times \lim\limits_{h \to 0} \frac{e^{-h}-1}{h}$

As one of the limits $\lim\limits_{h \to 0} \frac{e^{-h}-1}{h}$ can't be evaluated by directly putting the value of h as it will take 0/0 form.

So we need to take steps to find its value.

$\Rightarrow f'(x) = \lim\limits_{h \to 0} e^{-x} \times \lim\limits_{h \to 0} \frac{e^{-h}-1}{-h} \times (-1)$

Use the formula: $\lim\limits_{x \to 0} \frac{e^x-1}{x} = \log_e e = 1$

$\Rightarrow f'(x) = e^{-x} \times (-1)$

$\Rightarrow f'(x) = -e^{-x}$

Hence,

Derivative of $f(x) = e^{-x} = -e^{-x}$

2 B. Question

Differentiate the following from first principle.

e^{3x}

Answer

We need to find derivative of $f(x) = e^{3x}$

Derivative of a function f(x) is given by –

$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$ {where h is a very small positive number}

∴ derivative of $f(x) = e^{3x}$ is given as –

$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$

$\Rightarrow f'(x) = \lim_{h \to 0} \frac{e^{3(x+h)}-e^{3x}}{h}$

$\Rightarrow f'(x) = \lim_{h \to 0} \frac{e^{3x}e^{3h}-e^{3x}}{h}$

Taking e^{-x} common, we have –

$\Rightarrow f'(x) = \lim_{h \to 0} \frac{e^{3x}(e^{3h}-1)}{h}$

Using algebra of limits –

$\Rightarrow f'(x) = \lim_{h \to 0} e^{3x} \times \lim_{h \to 0} \frac{e^{3h}-1}{h}$

As one of the limits $\lim_{h \to 0} \frac{e^{3h}-1}{h}$ can't be evaluated by directly putting the value of h as it will take 0/0 form.

So we need to take steps to find its value.

$\Rightarrow f'(x) = \lim_{h \to 0} e^{3x} \times \lim_{h \to 0} \frac{e^{3h}-1}{3h} \times 3$

Use the formula: $\lim_{x \to 0} \frac{e^{x}-1}{x} = \log_e e = 1$

$\Rightarrow f'(x) = e^{3x} \times (3)$

$\Rightarrow f'(x) = 3e^{3x}$

Hence,

Derivative of $f(x) = e^{3x} = 3e^{3x}$

2 C. Question

Differentiate the following from first principle.

$e^{ax + b}$

Answer

We need to find derivative of $f(x) = e^{ax + b}$

Derivative of a function f(x) is given by –

$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$ {where h is a very small positive number}

∴ derivative of $f(x) = e^{ax + b}$ is given as –

$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$

$\Rightarrow f'(x) = \lim_{h \to 0} \frac{e^{a(x+h)+b} - e^{ax+b}}{h}$

$\Rightarrow f'(x) = \lim_{h \to 0} \frac{e^{ax+b}e^{ah} - e^{ax+b}}{h}$

Taking $e^{ax + b}$ common, we have –

$\Rightarrow f'(x) = \lim_{h \to 0} \frac{e^{ax+b}(e^{ah}-1)}{h}$

Using algebra of limits –

$\Rightarrow f'(x) = \lim_{h \to 0} e^{ax+b} \times \lim_{h \to 0} \frac{e^{ah}-1}{h}$

As one of the limits $\lim_{h \to 0} \frac{e^{ah}-1}{h}$ can't be evaluated by directly putting the value of h as it will take 0/0 form.

So we need to take steps to find its value.

$\Rightarrow f'(x) = \lim_{h \to 0} e^{ax+b} \times \lim_{h \to 0} \frac{e^{ah}-1}{ah} \times a$

Use the formula: $\lim_{x \to 0} \frac{e^x-1}{x} = \log_e e = 1$

$\Rightarrow f'(x) = e^{ax + b} \times (a)$

$\Rightarrow f'(x) = ae^{ax + b}$

Hence,

Derivative of $f(x) = e^{ax + b} = ae^{ax + b}$

2 D. Question

Differentiate the following from first principle.

xe^x

Answer

We need to find derivative of $f(x) = xe^x$

Derivative of a function f(x) is given by –

$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$ {where h is a very small positive number}

∴ derivative of $f(x) = xe^x$ is given as –

$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$

$\Rightarrow f'(x) = \lim_{h \to 0} \frac{(x+h)e^{(x+h)} - xe^x}{h}$

$\Rightarrow f'(x) = \lim_{h \to 0} \frac{he^{x+h} + xe^{x+h} - xe^x}{h}$

Using algebra of limits, we have –

$\Rightarrow f'(x) = \lim\limits_{h \to 0} \frac{he^{x+h}}{h} + \lim\limits_{h \to 0} \frac{x(e^{x+h} - e^{x})}{h}$

$\Rightarrow f'(x) = \lim\limits_{h \to 0} e^{x+h} + \lim\limits_{h \to 0} \frac{xe^{x}(e^{h} - 1)}{h}$

Again Using algebra of limits, we have –

$\Rightarrow f'(x) = e^{x+0} + \lim\limits_{h \to 0} \frac{(e^{h} - 1)}{h} \times \lim\limits_{h \to 0} xe^{x}$

Use the formula: $\lim\limits_{x \to 0} \frac{e^{x} - 1}{x} = \log_e e = 1$

$\Rightarrow f'(x) = e^x + xe^x$

$\Rightarrow f'(x) = e^x(x + 1)$

Hence,

Derivative of $f(x) = xe^x = e^x(x + 1)$

2 E. Question

Differentiate the following from first principle.

$-x$

Answer

We need to find derivative of $f(x) = -x$

Derivative of a function f(x) is given by –

$f'(x) = \lim\limits_{h \to 0} \frac{f(x+h) - f(x)}{h}$ {where h is a very small positive number}

∴ derivative of $f(x) = -x$is given as –

$f'(x) = \lim\limits_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$\Rightarrow f'(x) = \lim\limits_{h \to 0} \frac{-(x+h) - (-x)}{h}$

$\Rightarrow f'(x) = \lim\limits_{h \to 0} \frac{-x - h + x}{h}$

$\Rightarrow f'(x) = \lim\limits_{h \to 0} \frac{-h}{h} = \lim\limits_{h \to 0} -1$

∴ $f'(x) = -1$

Hence,

Derivative of $f(x) = -x = -1$

2 F. Question

Differentiate the following from first principle.

$(-x)^{-1}$

Answer

We need to find derivative of $f(x) = (-x)^{-1} = -1/x$

Derivative of a function f(x) is given by –

$f'(x) = \lim\limits_{h \to 0} \frac{f(x+h) - f(x)}{h}$ {where h is a very small positive number}

∴ derivative of $f(x) = -1/x$ is given as –

$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$

$\Rightarrow f'(x) = \lim_{h \to 0} \frac{\frac{-1}{x+h}-\left(-\frac{1}{x}\right)}{h} = \lim_{h \to 0} \frac{\frac{-1}{x+h}+\frac{1}{x}}{h}$

$\Rightarrow f'(x) = \lim_{h \to 0} \frac{\frac{-x+(x+h)}{x(x+h)}}{h} = \lim_{h \to 0} \frac{x-x+h}{h(x)(x+h)}$

$\Rightarrow f'(x) = \lim_{h \to 0} \frac{h}{h(x)(x+h)} = \lim_{h \to 0} \frac{1}{(x)(x+h)}$

As h is cancelled and by putting h = 0 we are not getting any indeterminate form so we can evaluate the limit directly.

$\therefore f'(x) = \frac{1}{x(x+0)} = \frac{1}{x^2}$

Hence,

Derivative of f(x) = $(-x)^{-1} = \frac{1}{x^2}$

2 G. Question

Differentiate the following from first principle.

sin (x + 1)

Answer

We need to find derivative of f(x) = sin (x + 1)

Derivative of a function f(x) is given by –

$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$ {where h is a very small positive number}

∴ derivative of f(x) = sin (x + 1) is given as –

$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$

$\Rightarrow f'(x) = \lim_{h \to 0} \frac{\sin(x+1+h)-\sin(x+1)}{h}$

We can't evaluate the limits at this stage only as on putting value it will take 0/0 form. So, we need to do little modifications.

Use: sin A – sin B = 2 cos ((A + B)/2) sin ((A – B)/2)

$\therefore f'(x) = \lim_{h \to 0} \frac{2\cos\left(\frac{2x+2+h}{2}\right)\sin\left(\frac{h}{2}\right)}{h}$

$\Rightarrow f'(x) = \lim_{h \to 0} \frac{\cos\left(x+1+\frac{h}{2}\right)\sin\left(\frac{h}{2}\right)}{\frac{h}{2}}$

Using algebra of limits –

$\Rightarrow f'(x) = \lim_{h \to 0} \frac{\sin\left(\frac{h}{2}\right)}{\frac{h}{2}} \times \lim_{h \to 0} \cos\left(x + 1 + \frac{h}{2}\right)$

Use the formula – $\lim_{x \to 0} \frac{\sin x}{x} = 1$

$\therefore f'(x) = 1 \times \lim_{h \to 0} \cos\left(x + 1 + \frac{h}{2}\right)$

Put the value of h to evaluate the limit –

∴ f'(x) = cos (x + 1 + 0) = cos (x + 1)

Hence,

Derivative of f(x) = sin (x + 1) = cos (x + 1)

2 H. Question

Differentiate the following from first principle.

$$\cos\left(x-\frac{\pi}{8}\right)$$

Answer

We need to find derivative of f(x) = cos (x – π/8)

Derivative of a function f(x) is given by –

f'(x) = $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ {where h is a very small positive number}

∴ derivative of f(x) = cos (x – π/8) is given as –

f'(x) = $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

⇒ f'(x) = $\lim_{h\to 0}\frac{\cos\left(x-\frac{\pi}{8}+h\right)-\cos\left(x-\frac{\pi}{8}\right)}{h}$

We can't evaluate the limits at this stage only as on putting value it will take 0/0 form. So, we need to do little modifications.

Use: cos A – cos B = – 2 sin ((A + B)/2) sin ((A – B)/2)

∴ f'(x) = $\lim_{h\to 0}\frac{-2\sin\left(\frac{2x-2\times\frac{\pi}{8}+h}{2}\right)\sin\left(\frac{h}{2}\right)}{h}$

⇒ f'(x) = $-\lim_{h\to 0}\frac{\sin\left(x-\frac{\pi}{8}+\frac{h}{2}\right)\sin\left(\frac{h}{2}\right)}{\frac{h}{2}}$

Using algebra of limits –

⇒ f'(x) = $-1\lim_{h\to 0}\frac{\sin\left(\frac{h}{2}\right)}{\frac{h}{2}}\times\lim_{h\to 0}\sin\left(x-\frac{\pi}{8}+\frac{h}{2}\right)$

Use the formula – $\lim_{x\to 0}\frac{\sin x}{x} = 1$

∴ f'(x) = $-1\times\lim_{h\to 0}\sin\left(x-\frac{\pi}{8}+\frac{h}{2}\right)$

Put the value of h to evaluate the limit –

∴ f'(x) = – sin (x – π/8 + 0) = – sin (x – π/8)

Hence,

Derivative of f(x) = cos (x – π/8) = – sin (x – π/8)

2 I. Question

Differentiate the following from first principle.

x sin x

Answer

We need to find derivative of f(x) = xsin x

Derivative of a function f(x) is given by –

$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$ {where h is a very small positive number}

∴ derivative of f(x) = x sin x is given as –

$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

$\Rightarrow f'(x) = \lim_{h\to 0} \frac{(x+h)\sin(x+h)-x\sin x}{h}$

$\Rightarrow f'(x) = \lim_{h\to 0} \frac{h\sin(x+h)+x\sin(x+h)-x\sin x}{h}$

Using algebra of limits, we have –

$\Rightarrow f'(x) = \lim_{h\to 0} \frac{h\sin(x+h)}{h} + \lim_{h\to 0} \frac{x\sin(x+h)-x\sin x}{h}$

$\Rightarrow f'(x) = \lim_{h\to 0} \sin(x+h) + \lim_{h\to 0} \frac{x(\sin(x+h)-\sin x)}{h}$

Using algebra of limits we have –

$\therefore f'(x) = \sin x + x \lim_{h\to 0} \frac{\sin(x+h)-\sin x}{h}$

We can’t evaluate the limits at this stage only as on putting value it will take 0/0 form. So, we need to do little modifications.

Use: sin A – sin B = 2 cos ((A + B)/2) sin ((A – B)/2)

$\therefore f'(x) = \sin x + x \lim_{h\to 0} \frac{2\cos\left(\frac{2x+h}{2}\right)\sin\left(\frac{h}{2}\right)}{h}$

$\Rightarrow f'(x) = \sin x + x \lim_{h\to 0} \frac{\cos\left(x+\frac{h}{2}\right)\sin\left(\frac{h}{2}\right)}{\frac{h}{2}}$

Using algebra of limits –

$\Rightarrow f'(x) = \sin x + x \lim_{h\to 0} \frac{\sin\left(\frac{h}{2}\right)}{\frac{h}{2}} \times \lim_{h\to 0} \cos\left(x + \frac{h}{2}\right)$

Use the formula – $\lim_{x\to 0} \frac{\sin x}{x} = 1$

$\therefore f'(x) = \sin x + x \times \lim_{h\to 0} \cos\left(x + \frac{h}{2}\right)$

Put the value of h to evaluate the limit –

∴ f’(x) = sin x + x cos(x + 0) = sin x + x cos x

Hence,

Derivative of f(x) = (x sin x) is (sin x + x cos x)

2 J. Question

Differentiate the following from first principle.

x cos x

Answer

We need to find derivative of f(x) = x cos x

Derivative of a function f(x) is given by –

$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$ {where h is a very small positive number}

∴ derivative of f(x) = x cos x is given as –

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\Rightarrow f'(x) = \lim_{h \to 0} \frac{(x+h)\cos(x+h) - x\cos(x)}{h}$$

$$\Rightarrow f'(x) = \lim_{h \to 0} \frac{h\cos(x+h) + x\cos(x+h) - x\cos x}{h}$$

Using algebra of limits, we have –

$$\Rightarrow f'(x) = \lim_{h \to 0} \frac{h\cos(x+h)}{h} + \lim_{h \to 0} \frac{x\cos(x+h) - x\cos x}{h}$$

$$\Rightarrow f'(x) = \lim_{h \to 0} \cos(x+h) + \lim_{h \to 0} \frac{x(\cos(x+h) - \cos x)}{h}$$

Using algebra of limits we have –

$$\therefore f'(x) = \cos x + x \lim_{h \to 0} \frac{\cos(x+h) - \cos x}{h}$$

We can't evaluate the limits at this stage only as on putting value it will take 0/0 form. So, we need to do little modifications.

Use: $\cos A - \cos B = -2 \sin((A + B)/2) \sin((A - B)/2)$

$$\therefore f'(x) = \cos x + x \lim_{h \to 0} \frac{-2\sin\left(\frac{2x+h}{2}\right)\sin\left(\frac{h}{2}\right)}{h}$$

$$\Rightarrow f'(x) = \cos x - x \lim_{h \to 0} \frac{\sin\left(\frac{2x+h}{2}\right)\sin\left(\frac{h}{2}\right)}{\frac{h}{2}}$$

Using algebra of limits –

$$\Rightarrow f'(x) = \cos x - x \lim_{h \to 0} \frac{\sin\left(\frac{h}{2}\right)}{\frac{h}{2}} \times \lim_{h \to 0} \sin\left(x + \frac{h}{2}\right)$$

Use the formula – $\lim_{x \to 0} \frac{\sin x}{x} = 1$

$$\therefore f'(x) = \cos x - x \times \lim_{h \to 0} \sin\left(x + \frac{h}{2}\right)$$

Put the value of h to evaluate the limit –

$\therefore f'(x) = \cos x - x \sin x$

Hence,

Derivative of $f(x) = x \cos x$ is $\cos x - x \sin x$

2 K. Question

Differentiate the following from first principle.

$\sin(2x - 3)$

Answer

We need to find derivative of $f(x) = \sin(2x - 3)$

Derivative of a function f(x) is given by –

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$ {where h is a very small positive number}

$\therefore$ derivative of $f(x) = \sin(2x - 3)$ is given as –

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$\Rightarrow f'(x) = \lim_{h \to 0} \frac{\sin(2(x+h)-3)-\sin(2x-3)}{h}$

We can't evaluate the limits at this stage only as on putting value it will take 0/0 form. So, we need to do little modifications.

Use: sin A – sin B = 2 cos ((A + B)/2) sin ((A – B)/2)

$\therefore f'(x) = \lim_{h \to 0} \frac{2\cos\left(\frac{4x-6+2h}{2}\right)\sin\left(\frac{2h}{2}\right)}{h}$

$\Rightarrow f'(x) = 2\lim_{h \to 0} \frac{\cos(2x-3+h)\sin(h)}{h}$

Using algebra of limits –

$\Rightarrow f'(x) = 2\lim_{h \to 0} \frac{\sin(h)}{h} \times \lim_{h \to 0} \cos(2x-3+h)$

Use the formula – $\lim_{x \to 0} \frac{\sin x}{x} = 1$

$\therefore f'(x) = 2 \times \lim_{h \to 0} \cos(2x-3+h)$

Put the value of h to evaluate the limit –

$\therefore f'(x) = 2\cos(2x-3+0) = 2\cos(2x-3)$

Hence,

Derivative of f(x) = sin (2x – 3) = 2cos (2x – 3)

3 A. Question

Differentiate the following from first principles

$\sqrt{\sin 2x}$

Answer

We need to find derivative of f(x) = √(sin 2x)

Derivative of a function f(x) is given by –

$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$ {where h is a very small positive number}

∴ derivative of f(x) = √(sin 2x) is given as –

$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$

$\Rightarrow f'(x) = \lim_{h \to 0} \frac{\sqrt{\sin 2(x+h)}-\sqrt{\sin 2x}}{h}$

We can't evaluate the limits at this stage only as on putting value it will take 0/0 form. So, we need to do little modifications.

Multiplying numerator and denominator by √(sin 2(x + h)) + √(sin 2x), we have –

$\Rightarrow f'(x) = \lim_{h \to 0} \frac{\sqrt{\sin 2(x+h)}-\sqrt{\sin 2x}}{h} \times \frac{\sqrt{\sin 2(x+h)}+\sqrt{\sin 2x}}{\sqrt{\sin 2(x+h)}+\sqrt{\sin 2x}}$

Using $a^2 - b^2 = (a+b)(a-b)$, we have –

$\Rightarrow f'(x) = \lim_{h \to 0} \frac{\left(\sqrt{\sin 2(x+h)}\right)^2 - \left(\sqrt{\sin 2x}\right)^2}{h\sqrt{\sin 2(x+h)}+\sqrt{\sin 2x}}$

Again using algebra of limits, we get –

$$\Rightarrow f'(x) = \lim_{h\to 0} \frac{\sin(2x+2h)-\sin 2x}{h} \times \lim_{h\to 0} \frac{1}{\sqrt{\sin 2(x+h)}+\sqrt{\sin 2x}}$$

Use: sin A – sin B = 2 cos ((A + B)/2) sin ((A – B)/2)

$$\therefore f'(x) = \frac{1}{2\sqrt{\sin 2x}} \lim_{h\to 0} \frac{2\cos\left(\frac{4x+2h}{2}\right)\sin\left(\frac{2h}{2}\right)}{h}$$

$$\Rightarrow f'(x) = \frac{1}{\sqrt{\sin 2x}} \lim_{h\to 0} \frac{\cos(2x+h)\sin(h)}{h}$$

Using algebra of limits –

$$\Rightarrow f'(x) = \frac{1}{\sqrt{\sin 2x}} \lim_{h\to 0} \frac{\sin(h)}{h} \times \lim_{h\to 0} \cos(2x + h)$$

Use the formula – $\lim_{x\to 0} \frac{\sin x}{x} = 1$

$$\therefore f'(x) = \frac{1}{\sqrt{\sin 2x}} \times \lim_{h\to 0} \cos(2x + h)$$

Put the value of h to evaluate the limit –

$$\therefore f'(x) = \frac{\cos 2x}{\sqrt{\sin 2x}}$$

Hence,

Derivative of f(x) = √(sin 2x) = $\frac{\cos 2x}{\sqrt{\sin 2x}}$

3 B. Question

Differentiate the following from first principles

$$\frac{\sin x}{x}$$

Answer

We need to find derivative of f(x) = (sin x)/x

Derivative of a function f(x) is given by –

$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$ {where h is a very small positive number}

∴ derivative of f(x) = (sin x)/x is given as –

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$\Rightarrow f'(x) = \lim_{h\to 0} \frac{\frac{\sin(x+h)}{x+h}-\frac{\sin x}{x}}{h}$$

$$\Rightarrow f'(x) = \lim_{h\to 0} \frac{\frac{x\sin(x+h)-(x+h)\sin x}{x(x+h)}}{h} = \lim_{h\to 0} \frac{x\sin(x+h)-(x+h)\sin x}{h(x)(x+h)}$$

Using algebra of limits we have –

$$\therefore f'(x) = \lim_{h\to 0} \frac{x\sin(x+h)-(x+h)\sin x}{h} \times \lim_{h\to 0} \frac{1}{x(x+h)}$$

$$\Rightarrow f'(x) = \lim_{h\to 0} \frac{x\sin(x+h)-(x+h)\sin x}{h} \times \frac{1}{x(x+0)}$$

$$\Rightarrow f'(x) = \frac{1}{x^2} \lim_{h\to 0} \frac{x\sin(x+h)-(x+h)\sin x}{h}$$

$$\Rightarrow f'(x) = \frac{1}{x^2} \lim_{h\to 0} \frac{x\sin(x+h)-x\sin x-h\sin x}{h}$$

Using algebra of limits, we have –

$\Rightarrow f'(x) = \frac{1}{x^2}\left\{\lim_{h\to 0}\frac{-h\sin x}{h} + \lim_{h\to 0}\frac{x\sin(x+h)-x\sin x}{h}\right\}$

$\Rightarrow f'(x) = \frac{1}{x^2}\left\{-\lim_{h\to 0}\sin x + \lim_{h\to 0}\frac{x(\sin(x+h)-\sin x)}{h}\right\}$

Using algebra of limits we have –

$\therefore f'(x) = -\frac{\sin x}{x^2} + \frac{1}{x}\lim_{h\to 0}\frac{\sin(x+h)-\sin x}{h}$

We can't evaluate the limits at this stage only as on putting value it will take 0/0 form. So, we need to do little modifications.

Use: sin A – sin B = 2 cos ((A + B)/2) sin ((A – B)/2)

$\therefore f'(x) = -\frac{\sin x}{x^2} + \frac{1}{x}\lim_{h\to 0}\frac{2\cos\left(\frac{2x+h}{2}\right)\sin\left(\frac{h}{2}\right)}{h}$

$\Rightarrow f'(x) = -\frac{\sin x}{x^2} + \frac{1}{x}\lim_{h\to 0}\frac{\cos\left(x+\frac{h}{2}\right)\sin\left(\frac{h}{2}\right)}{\frac{h}{2}}$

Using algebra of limits –

$\Rightarrow f'(x) = -\frac{\sin x}{x^2} + \frac{1}{x}\lim_{h\to 0}\frac{\sin\left(\frac{h}{2}\right)}{\frac{h}{2}} \times \lim_{h\to 0}\cos\left(x + \frac{h}{2}\right)$

Use the formula – $\lim_{x\to 0}\frac{\sin x}{x} = 1$

$\therefore f'(x) = -\frac{\sin x}{x^2} + \frac{1}{x} \times \lim_{h\to 0}\cos\left(x + \frac{h}{2}\right)$

Put the value of h to evaluate the limit –

$\therefore f'(x) = -\frac{\sin x}{x^2} + \frac{1}{x} \times \cos(x + 0) = -\frac{\sin x}{x^2} + \frac{\cos x}{x}$

Hence,

Derivative of f(x) = (sin x)/x is $-\frac{\sin x}{x^2} + \frac{\cos x}{x}$

3 C. Question

Differentiate the following from first principles

$\frac{\cos x}{x}$

Answer

We need to find derivative of f(x) = (cos x)/x

Derivative of a function f(x) is given by –

$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ {where h is a very small positive number}

∴ derivative of f(x) = (cos x)/x is given as –

$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$\Rightarrow f'(x) = \lim_{h\to 0}\frac{\frac{\cos(x+h)}{x+h}-\frac{\cos x}{x}}{h}$

$\Rightarrow f'(x) = \lim_{h\to 0}\frac{\frac{x\cos(x+h)-(x+h)\cos x}{x(x+h)}}{h} = \lim_{h\to 0}\frac{x\cos(x+h)-(x+h)\cos x}{h(x)(x+h)}$

Using algebra of limits we have –

$$\therefore f'(x) = \lim_{h \to 0} \frac{x\cos(x+h)-(x+h)\cos x}{h} \times \lim_{h \to 0} \frac{1}{x(x+h)}$$

$$\Rightarrow f'(x) = \lim_{h \to 0} \frac{x\cos(x+h)-(x+h)\cos x}{h} \times \frac{1}{x(x+0)}$$

$$\Rightarrow f'(x) = \frac{1}{x^2} \lim_{h \to 0} \frac{x\cos(x+h)-(x+h)\cos x}{h}$$

$$\Rightarrow f'(x) = \frac{1}{x^2} \lim_{h \to 0} \frac{x\cos(x+h)-x\cos x-h\cos x}{h}$$

Using algebra of limits, we have –

$$\Rightarrow f'(x) = \frac{1}{x^2}\left\{\lim_{h \to 0} \frac{-h\cos x}{h} + \lim_{h \to 0} \frac{x\cos(x+h)-x\cos x}{h}\right\}$$

$$\Rightarrow f'(x) = \frac{1}{x^2}\left\{-\lim_{h \to 0} \cos x + \lim_{h \to 0} \frac{x(\cos(x+h)-\cos x)}{h}\right\}$$

Using algebra of limits we have –

$$\therefore f'(x) = -\frac{\cos x}{x^2} + \frac{1}{x} \lim_{h \to 0} \frac{\cos(x+h)-\cos x}{h}$$

We can't evaluate the limits at this stage only as on putting value it will take 0/0 form. So, we need to do little modifications.

Use: cos A – cos B = – 2 sin ((A + B)/2) sin ((A – B)/2)

$$\therefore f'(x) = -\frac{\cos x}{x^2} + \frac{1}{x} \lim_{h \to 0} \frac{-2\sin\left(\frac{2x+h}{2}\right)\sin\left(\frac{h}{2}\right)}{h}$$

$$\Rightarrow f'(x) = -\frac{\cos x}{x^2} - \frac{1}{x} \lim_{h \to 0} \frac{\sin\left(x+\frac{h}{2}\right)\sin\left(\frac{h}{2}\right)}{\frac{h}{2}}$$

Using algebra of limits –

$$\Rightarrow f'(x) = -\frac{\cos x}{x^2} - \frac{1}{x} \lim_{h \to 0} \frac{\sin\left(\frac{h}{2}\right)}{\frac{h}{2}} \times \lim_{h \to 0} \sin\left(x + \frac{h}{2}\right)$$

Use the formula – $\lim_{x \to 0} \frac{\sin x}{x} = 1$

$$\therefore f'(x) = -\frac{\cos x}{x^2} - \frac{1}{x} \times \lim_{h \to 0} \sin\left(x + \frac{h}{2}\right)$$

Put the value of h to evaluate the limit –

$$\therefore f'(x) = -\frac{\cos x}{x^2} - \frac{1}{x} \times \sin(x + 0) = -\frac{\cos x}{x^2} - \frac{\sin x}{x}$$

Hence,

Derivative of f(x) = (cos x)/x is $-\frac{\cos x}{x^2} - \frac{\sin x}{x}$

3 D. Question

Differentiate the following from first principles

$x^2 \sin x$

Answer

We need to find derivative of f(x) = x^2 sin x

Derivative of a function f(x) is given by –

$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$ {where h is a very small positive number}

∴ derivative of $f(x) = x^2 \sin x$ is given as –

$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$

$\Rightarrow f'(x) = \lim_{h \to 0} \frac{(x+h)^2 \sin(x+h) - x^2 \sin x}{h}$

Using $(a + b)^2 = a^2 + 2ab + b^2$,we have –

$\Rightarrow f'(x) = \lim_{h \to 0} \frac{h^2 \sin(x+h) + x^2 \sin(x+h) + 2hx\sin(x+h) - x^2 \sin x}{h}$

Using algebra of limits, we have –

$\Rightarrow f'(x) = \lim_{h \to 0} \frac{h^2 \sin(x+h)}{h} + \lim_{h \to 0} \frac{x^2 \sin(x+h) - x^2 \sin x}{h} + \lim_{h \to 0} \frac{2hx\sin(x+h)}{h}$

$\Rightarrow f'(x) = \lim_{h \to 0} h \sin(x + h) + \lim_{h \to 0} \frac{x^2(\sin(x+h) - \sin x)}{h} + \lim_{h \to 0} 2x \sin(x + h)$

$\Rightarrow f'(x) = 0 \times \sin(x + 0) + 2x \sin(x + 0) + x^2 \lim_{h \to 0} \frac{(\sin(x+h) - \sin x)}{h}$

$\Rightarrow f'(x) = 2x \sin x + x^2 \lim_{h \to 0} \frac{(\sin(x+h) - \sin x)}{h}$

Using algebra of limits we have –

$\therefore f'(x) = 2x \sin x + x^2 \lim_{h \to 0} \frac{\sin(x+h) - \sin x}{h}$

We can’t evaluate the limits at this stage only as on putting value it will take 0/0 form. So, we need to do little modifications.

Use: sin A – sin B = 2 cos ((A + B)/2) sin ((A – B)/2)

$\therefore f'(x) = 2x \sin x + x^2 \lim_{h \to 0} \frac{2 \cos\left(\frac{2x+h}{2}\right) \sin\left(\frac{h}{2}\right)}{h}$

$\Rightarrow f'(x) = 2x \sin x + x^2 \lim_{h \to 0} \frac{\cos\left(x + \frac{h}{2}\right) \sin\left(\frac{h}{2}\right)}{\frac{h}{2}}$

Using algebra of limits –

$\Rightarrow f'(x) = 2x \sin x + x^2 \lim_{h \to 0} \frac{\sin\left(\frac{h}{2}\right)}{\frac{h}{2}} \times \lim_{h \to 0} \cos\left(x + \frac{h}{2}\right)$

Use the formula – $\lim_{x \to 0} \frac{\sin x}{x} = 1$

$\therefore f'(x) = 2x \sin x + x^2 \times \lim_{h \to 0} \cos\left(x + \frac{h}{2}\right)$

Put the value of h to evaluate the limit –

$\therefore f'(x) = 2x \sin x + x^2 \cos(x + 0) = 2x \sin x + x^2 \cos x$

Hence,

Derivative of $f(x) = (x^2 \sin x)$ is $(2x \sin x + x^2 \cos x)$

3 E. Question

Differentiate the following from first principles

$\sqrt{\sin(3x+1)}$

Answer

We need to find derivative of f(x) = √(sin (3x + 1))

Derivative of a function f(x) is given by –

f'(x) = $\lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$ {where h is a very small positive number}

∴ derivative of f(x) = √(sin (3x + 1)) is given as –

f'(x) = $\lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$

⇒ f'(x) = $\lim_{h \to 0} \frac{\sqrt{\sin\{3(x+h)+1\}}-\sqrt{\sin(3x+1)}}{h}$

We can't evaluate the limits at this stage only as on putting value it will take 0/0 form. So, we need to do little modifications.

Multiplying numerator and denominator by $\sqrt{\sin\{3(x + h) + 1\}} + \sqrt{\sin(3x + 1)}$, we have –

⇒ f'(x) = $\lim_{h \to 0} \frac{\sqrt{\sin\{3(x+h)+1\}}-\sqrt{\sin(3x+1)}}{h} \times \frac{\sqrt{\sin\{3(x+h)+1\}}+\sqrt{\sin(3x+1)}}{\sqrt{\sin\{3(x+h)+1\}}+\sqrt{\sin(3x+1)}}$

Using $a^2 - b^2 = (a + b)(a - b)$, we have –

⇒ f'(x) = $\lim_{h \to 0} \frac{\left(\sqrt{\sin\{3(x+h)+1\}}\right)^2-\left(\sqrt{\sin(3x+1)}\right)^2}{h\sqrt{\sin\{3(x+h)+1\}}+\sqrt{\sin(3x+1)}}$

Again using algebra of limits, we get –

⇒ f'(x) = $\lim_{h \to 0} \frac{\sin(3x+3h+1)-\sin(3x+1)}{h} \times \lim_{h \to 0} \frac{1}{\sqrt{\sin\{3(x+h)+1\}}+\sqrt{\sin(3x+1)}}$

Use: sin A – sin B = 2 cos ((A + B)/2) sin ((A – B)/2)

∴ f'(x) = $\frac{1}{2\sqrt{\sin(3x+1)}} \lim_{h \to 0} \frac{2\cos\left(\frac{6x+3h+2}{2}\right)\sin\left(\frac{3h}{2}\right)}{h}$

⇒ f'(x) = $\frac{1}{\sqrt{\sin(3x+1)}} \lim_{h \to 0} \frac{\cos\left(3x+1+\frac{3h}{2}\right)\sin\left(\frac{3h}{2}\right)}{h}$

Using algebra of limits –

⇒ f'(x) = $\frac{1}{\sqrt{\sin(3x+1)}} \lim_{h \to 0} \frac{\frac{3}{2}\sin\left(\frac{3h}{2}\right)}{\frac{3h}{2}} \times \lim_{h \to 0} \cos\left(3x + 1 + \frac{3h}{2}\right)$

Use the formula – $\lim_{x \to 0} \frac{\sin x}{x} = 1$

∴ f'(x) = $\frac{3}{2\sqrt{\sin(3x+1)}} \times \lim_{h \to 0} \cos\left(3x + 1 + \frac{h}{2}\right)$

Put the value of h to evaluate the limit –

∴ f'(x) = $\frac{3\cos(3x+1)}{2\sqrt{\sin(3x+1)}}$

Hence,

Derivative of f(x) = √(sin (3x + 1)) = $\frac{3\cos(3x+1)}{2\sqrt{\sin(3x+1)}}$

3 F. Question

Differentiate the following from first principles

sin x + cos x

Answer

We need to find derivative of f(x) = sin x + cos x

Derivative of a function f(x) is given by –

$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$ {where h is a very small positive number}

∴ derivative of f(x) = sin x + cos x is given as –

$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$

$\Rightarrow f'(x) = \lim_{h \to 0} \frac{\sin(x+h) + \cos(x+h) - (\sin x + \cos x)}{h}$

Using algebra of limits we have –

$\Rightarrow f'(x) = \lim_{h \to 0} \frac{\sin(x+h) - \sin x}{h} + \lim_{h \to 0} \frac{\cos(x+h) - \cos x}{h}$

We can't evaluate the limits at this stage only as on putting value it will take 0/0 form. So, we need to do little modifications.

Use: sin A – sin B = 2 cos ((A + B)/2) sin ((A – B)/2) and

cos A – cos B = – 2 sin ((A + B)/2) sin ((A – B)/2)

$\therefore f'(x) = \lim_{h \to 0} \frac{2\cos\left(\frac{2x+h}{2}\right)\sin\left(\frac{h}{2}\right)}{h} + \lim_{h \to 0} \frac{-2\sin\left(\frac{2x+h}{2}\right)\sin\left(\frac{h}{2}\right)}{h}$

Dividing numerator and denominator by 2 in both the terms –

$\Rightarrow f'(x) = \lim_{h \to 0} \frac{\cos\left(x+\frac{h}{2}\right)\sin\left(\frac{h}{2}\right)}{\frac{h}{2}} - \lim_{h \to 0} \frac{\sin\left(x+\frac{h}{2}\right)\sin\left(\frac{h}{2}\right)}{\frac{h}{2}}$

Using algebra of limits –

$\Rightarrow f'(x) = \lim_{h \to 0} \frac{\sin\left(\frac{h}{2}\right)}{\frac{h}{2}} \times \lim_{h \to 0} \cos\left(x + \frac{h}{2}\right) - \lim_{h \to 0} \frac{\sin\left(\frac{h}{2}\right)}{\frac{h}{2}} \times \lim_{h \to 0} \sin\left(x + \frac{h}{2}\right)$

Use the formula – $\lim_{x \to 0} \frac{\sin x}{x} = 1$

$\therefore f'(x) = 1 \times \lim_{h \to 0} \cos\left(x + \frac{h}{2}\right) - 1 \times \lim_{h \to 0} \sin\left(x + \frac{h}{2}\right)$

Put the value of h to evaluate the limit –

∴ f'(x) = cos (x + 0) – sin (x + 0) = cos x – sin x

Hence,

Derivative of f(x) = sin x + cos x = cos x – sin x

3 G. Question

Differentiate the following from first principles

$x^2 e^x$

Answer

We need to find derivative of f(x) = x^2 e^x

Derivative of a function f(x) is given by –

$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$ {where h is a very small positive number}

∴ derivative of f(x) = x^2 e^x is given as –

$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$

$\Rightarrow f'(x) = \lim_{h \to 0} \frac{(x+h)^2 e^{(x+h)} - x^2 e^x}{h}$

$\Rightarrow f'(x) = \lim_{h \to 0} \frac{h^2 e^{x+h} + x^2 e^{x+h} + 2hxe^{x+h} - x^2 e^x}{h}$

Using algebra of limits, we have –

$\Rightarrow f'(x) = \lim_{h \to 0} \frac{h^2 e^{x+h}}{h} + \lim_{h \to 0} \frac{x^2 (e^{x+h} - e^x)}{h} + \lim_{h \to 0} \frac{2hxe^{x+h}}{h}$

$\Rightarrow \lim_{h \to 0} he^{x+h} + x^2 \lim_{h \to 0} \frac{e^x (e^h - 1)}{h} + \lim_{h \to 0} 2xe^{x+h}$

As 2 of the terms will not take indeterminate form if we put value of h = 0, so obtained their limiting value as follows –

$\therefore f'(x) = 0 \times e^{x+0} + 2x\, e^{x+0} + e^x x^2 \lim_{h \to 0} \frac{(e^h - 1)}{h}$

Use the formula: $\lim_{x \to 0} \frac{e^x - 1}{x} = \log_e e = 1$

$\Rightarrow f'(x) = 2x\, e^x + x^2\, e^x$

$\Rightarrow f'(x) = 2x\, e^x + x^2\, e^x$

Hence,

Derivative of $f(x) = x^2\, e^x = 2x\, e^x + x^2\, e^x$

3 H. Question

Differentiate the following from first principles

$e^{x^2} + 1$

Answer

We need to find derivative of $f(x) = e^{x^2} + 1$

Derivative of a function f(x) is given by –

$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$ {where h is a very small positive number}

$\therefore$ derivative of $f(x) = e^{x^2} + 1$ is given as –

$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$

$\Rightarrow f'(x) = \lim_{h \to 0} \frac{e^{(x+h)^2} + 1 - e^{x^2} - 1}{h}$

$\Rightarrow f'(x) = \lim_{h \to 0} \frac{e^{x^2 + 2hx + h^2} - e^{x^2}}{h}$

Taking e^{x^2} common, we have –

$\Rightarrow f'(x) = \lim_{h \to 0} \frac{e^{x^2} (e^{2hx + h^2} - 1)}{h}$

Using algebra of limits –

$\Rightarrow f'(x) = \lim_{h \to 0} e^{x^2} \times \lim_{h \to 0} \frac{e^{2hx + h^2} - 1}{h}$

$\Rightarrow f'(x) = e^{x^2} \lim_{h \to 0} \frac{e^{2hx+h^2}-1}{h}$

As one of the limits $\lim_{h \to 0} \frac{e^{2hx+h^2}-1}{h}$ can't be evaluated by directly putting the value of h as it will take 0/0 form.

So we need to take steps to find its value.

As h → 0 so, $(2hx + h^2) \to 0$

∴ multiplying numerator and denominator by $(2hx + h^2)$ in order to apply the formula –

$\lim_{x \to 0} \frac{e^x-1}{x} = \log_e e = 1$

$\therefore f'(x) = e^{x^2} \lim_{h \to 0} \frac{e^{2hx+h^2}-1}{2hx+h^2} \times \frac{2hx+h^2}{h}$

Again using algebra of limits, we have –

$\Rightarrow f'(x) = e^{x^2} \lim_{h \to 0} \frac{e^{2hx+h^2}-1}{2hx+h^2} \times \lim_{h \to 0} \frac{h(2x+h)}{h}$

Use the formula: $\lim_{x \to 0} \frac{e^x-1}{x} = \log_e e = 1$

$\Rightarrow f'(x) = e^{x^2} \lim_{h \to 0} \frac{h(2x+h)}{h}$

$\Rightarrow f'(x) = e^{x^2} \lim_{h \to 0} (2x + h)$

$\therefore f'(x) = e^{x^2} \times (2x + 0) = 2x\, e^{x^2}$

Hence,

Derivative of f(x) = $e^{x^2} + 1 = 2x\, e^{x^2}$

3 I. Question

Differentiate the following from first principles

$e^{\sqrt{2x}}$

Answer

We need to find derivative of f(x) = $e^{\sqrt{2x}}$

Derivative of a function f(x) is given by –

$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$ {where h is a very small positive number}

∴ derivative of f(x) = $e^{\sqrt{2x}}$ is given as –

$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$

$\Rightarrow f'(x) = \lim_{h \to 0} \frac{e^{\sqrt{2(x+h)}}-e^{\sqrt{2x}}}{h}$

$\Rightarrow f'(x) = \lim_{h \to 0} \frac{e^{\sqrt{2x+2h}}-e^{\sqrt{2x}}}{h}$

Taking $e^{\sqrt{2x}}$ common, we have –

$\Rightarrow f'(x) = \lim_{h \to 0} \frac{e^{\sqrt{2x}}(e^{\sqrt{2x+2h}-\sqrt{2x}}-1)}{h}$

Using algebra of limits –

$\Rightarrow f'(x) = \lim_{h \to 0} e^{\sqrt{2x}} \times \lim_{h \to 0} \frac{(e^{\sqrt{2x+2h}-\sqrt{2x}}-1)}{h}$

$\Rightarrow f'(x) = e^{\sqrt{2x}} \lim_{h \to 0} \frac{(e^{\sqrt{2x+2h}-\sqrt{2x}}-1)}{h}$

As one of the limits $\lim_{h \to 0} \frac{(e^{\sqrt{2x+2h}-\sqrt{2x}}-1)}{h}$ can't be evaluated by directly putting the value of h as it will take 0/0 form.

So we need to take steps to find its value.

As $h \to 0$ so, $(\sqrt{2x + 2h} - \sqrt{2x}) \to 0$

∴ multiplying numerator and denominator by $\sqrt{2x + 2h} - \sqrt{2x}$ in order to apply the formula – $\lim_{x \to 0} \frac{e^x-1}{x} = \log_e e = 1$

$\therefore f'(x) = e^{\sqrt{2x}} \lim_{h \to 0} \frac{e^{\sqrt{2x+2h}-\sqrt{2x}}-1}{\sqrt{2x+2h}-\sqrt{2x}} \times \frac{\sqrt{2x+2h}-\sqrt{2x}}{h}$

Again using algebra of limits, we have –

$\Rightarrow f'(x) = e^{\sqrt{2x}} \lim_{h \to 0} \frac{e^{\sqrt{2x+2h}-\sqrt{2x}}-1}{\sqrt{2x+2h}-\sqrt{2x}} \times \lim_{h \to 0} \frac{\sqrt{2x+2h}-\sqrt{2x}}{h}$

Use the formula: $\lim_{x \to 0} \frac{e^x-1}{x} = \log_e e = 1$

$\Rightarrow f'(x) = e^{\sqrt{2x}} \lim_{h \to 0} \frac{\sqrt{2x+2h}-\sqrt{2x}}{h}$

Again we get an indeterminate form, so multiplying and dividing √(2x + 2h) + √(2x) to get rid of indeterminate form.

$\therefore f'(x) = e^{\sqrt{2x}} \lim_{h \to 0} \frac{\sqrt{2x+2h}-\sqrt{2x}}{h} \times \frac{\sqrt{2x+2h}+\sqrt{2x}}{\sqrt{2x+2h}+\sqrt{2x}}$

Using $a^2 - b^2 = (a + b)(a - b)$, we have –

$\Rightarrow f'(x) = e^{\sqrt{2x}} \lim_{h \to 0} \frac{(\sqrt{2x+2h})^2-(\sqrt{2x})^2}{h(\sqrt{2x+2h}+\sqrt{2x})}$

Using algebra of limits we have –

$\Rightarrow f'(x) = e^{\sqrt{2x}} \lim_{h \to 0} \frac{2x+2h-2h}{h} \times \lim_{h \to 0} \frac{1}{\sqrt{2x+2h}+\sqrt{2x}}$

$\Rightarrow f'(x) = e^{\sqrt{2x}} \lim_{h \to 0} \frac{2h}{h} \times \frac{1}{\sqrt{2x+2(0)}+\sqrt{2x}}$

$\Rightarrow f'(x) = e^{\sqrt{2x}} \lim_{h \to 0} 2 \times \frac{1}{2\sqrt{2x}}$

$\therefore f'(x) = \frac{e^{\sqrt{2x}}}{\sqrt{2x}}$

Hence,

Derivative of $f(x) = e^{\sqrt{2x}} = \frac{e^{\sqrt{2x}}}{\sqrt{2x}}$

3 J. Question

Differentiate the following from first principles

$e^{\sqrt{ax+b}}$

Answer

We need to find derivative of f(x) = $e^{\sqrt{(ax + b)}}$

Derivative of a function f(x) is given by –

$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$ {where h is a very small positive number}

∴ derivative of f(x) = $e^{\sqrt{(ax + b)}}$ is given as –

$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$

$\Rightarrow f'(x) = \lim_{h \to 0} \frac{e^{\sqrt{a(x+h)+b}} - e^{\sqrt{ax+b}}}{h}$

$\Rightarrow f'(x) = \lim_{h \to 0} \frac{e^{\sqrt{ax+ah+b}} - e^{\sqrt{ax+b}}}{h}$

Taking $e^{\sqrt{ax+b}}$ common, we have –

$\Rightarrow f'(x) = \lim_{h \to 0} \frac{e^{\sqrt{ax+b}}\left(e^{\sqrt{ax+ah+b}-\sqrt{ax+b}} - 1\right)}{h}$

Using algebra of limits –

$\Rightarrow f'(x) = \lim_{h \to 0} e^{\sqrt{ax+b}} \times \lim_{h \to 0} \frac{\left(e^{\sqrt{ax+ah+b}-\sqrt{ax+b}} - 1\right)}{h}$

$\Rightarrow f'(x) = e^{\sqrt{ax+b}} \lim_{h \to 0} \frac{\left(e^{\sqrt{ax+ah+b}-\sqrt{ax+b}} - 1\right)}{h}$

As one of the limits $\lim_{h \to 0} \frac{\left(e^{\sqrt{ax+ah+b}-\sqrt{ax+b}} - 1\right)}{h}$ can't be evaluated by directly putting the value of h as it will take 0/0 form.

So we need to take steps to find its value.

As h → 0 so, $(\sqrt{ax + ah + b} - \sqrt{ax + b}) \to 0$

∴ multiplying numerator and denominator by $\sqrt{ax + ah + b} - \sqrt{ax + b}$ in order to apply the formula –

$\lim_{x \to 0} \frac{e^x - 1}{x} = \log_e e = 1$

$\therefore f'(x) = e^{\sqrt{ax+b}} \lim_{h \to 0} \frac{e^{\sqrt{ax+ah+b}-\sqrt{ax+b}} - 1}{\sqrt{ax+ah+b}-\sqrt{ax+b}} \times \frac{\sqrt{ax+ah+b}-\sqrt{ax+b}}{h}$

Again using algebra of limits, we have –

$\Rightarrow f'(x) = e^{\sqrt{ax+b}} \lim_{h \to 0} \frac{e^{\sqrt{ax+ah+b}-\sqrt{ax+b}} - 1}{\sqrt{ax+ah+b}-\sqrt{ax+b}} \times \lim_{h \to 0} \frac{\sqrt{ax+ah+b}-\sqrt{ax+b}}{h}$

Use the formula: $\lim_{x \to 0} \frac{e^x - 1}{x} = \log_e e = 1$

$\Rightarrow f'(x) = e^{\sqrt{ax+b}} \lim_{h \to 0} \frac{\sqrt{ax+ah+b}-\sqrt{ax+b}}{h}$

Again we get an indeterminate form, so multiplying and dividing √(ax + ah + b) + √(ax + b) to get rid of indeterminate form.

$\therefore f'(x) = e^{\sqrt{ax+b}} \lim_{h \to 0} \frac{\sqrt{ax+ah+b}-\sqrt{ax+b}}{h} \times \frac{\sqrt{ax+ah+b}+\sqrt{ax+b}}{\sqrt{ax+ah+b}+\sqrt{ax+b}}$

Using $a^2 - b^2 = (a + b)(a - b)$, we have –

$\Rightarrow f'(x) = e^{\sqrt{ax+b}} \lim_{h \to 0} \frac{\left(\sqrt{ax+ah+b}\right)^2 - \left(\sqrt{ax+b}\right)^2}{h\left(\sqrt{ax+ah+b}+\sqrt{ax+b}\right)}$

Using algebra of limits we have –

$\Rightarrow f'(x) = e^{\sqrt{ax+b}} \lim\limits_{h \to 0} \frac{ax+ah+b-ax-b}{h} \times \lim\limits_{h \to 0} \frac{1}{\sqrt{ax+ah+b}+\sqrt{ax+b}}$

$\Rightarrow f'(x) = e^{\sqrt{ax+b}} \lim\limits_{h \to 0} \frac{ah}{h} \times \frac{1}{\sqrt{ax+a(0)+b}+\sqrt{ax+b}}$

$\Rightarrow f'(x) = e^{\sqrt{ax+b}} \lim\limits_{h \to 0} a \times \frac{1}{2\sqrt{ax+b}}$

$\therefore f'(x) = \frac{ae^{\sqrt{ax+b}}}{2\sqrt{ax+b}}$

Hence,

Derivative of $f(x) = e^{\sqrt{(ax+b)}} = \frac{ae^{\sqrt{ax+b}}}{2\sqrt{ax+b}}$

3 K. Question

Differentiate the following from first principles

$a^{\sqrt{x}}$

Answer

We need to find derivative of $f(x) = a^{\sqrt{x}}$

Derivative of a function f(x) is given by –

$f'(x) = \lim\limits_{h \to 0} \frac{f(x+h)-f(x)}{h}$ {where h is a very small positive number}

$\therefore$ derivative of $f(x) = a^{\sqrt{x}}$ is given as –

$f'(x) = \lim\limits_{h \to 0} \frac{f(x+h)-f(x)}{h}$

$\Rightarrow f'(x) = \lim\limits_{h \to 0} \frac{a^{\sqrt{(x+h)}}-a^{\sqrt{x}}}{h}$

$\Rightarrow f'(x) = \lim\limits_{h \to 0} \frac{a^{\sqrt{x+h}}-a^{\sqrt{x}}}{h}$

Taking $a^{\sqrt{x}}$ common, we have –

$\Rightarrow f'(x) = \lim\limits_{h \to 0} \frac{a^{\sqrt{x}}(a^{\sqrt{x+h}-\sqrt{x}}-1)}{h}$

Using algebra of limits –

$\Rightarrow f'(x) = \lim\limits_{h \to 0} a^{\sqrt{x}} \times \lim\limits_{h \to 0} \frac{(a^{\sqrt{x+h}-\sqrt{x}}-1)}{h}$

$\Rightarrow f'(x) = a^{\sqrt{x}} \lim\limits_{h \to 0} \frac{(a^{\sqrt{x+h}-\sqrt{x}}-1)}{h}$

As one of the limits $\lim\limits_{h \to 0} \frac{(a^{\sqrt{x+h}-\sqrt{x}}-1)}{h}$ can't be evaluated by directly putting the value of h as it will take 0/0 form.

So we need to take steps to find its value.

As $h \to 0$ so, $(\sqrt{x+h}-\sqrt{x}) \to 0$

$\therefore$ multiplying numerator and denominator by $\sqrt{x+h}-\sqrt{x}$ in order to apply the formula – $\lim\limits_{x \to 0} \frac{a^x-1}{x} = \log_e a$

$\therefore f'(x) = a^{\sqrt{x}} \lim\limits_{h \to 0} \frac{a^{\sqrt{x+h}-\sqrt{x}}-1}{\sqrt{x+h}-\sqrt{x}} \times \frac{\sqrt{x+h}-\sqrt{x}}{h}$

Again using algebra of limits, we have –

$\Rightarrow f'(x) = a^{\sqrt{x}} \lim\limits_{h \to 0} \frac{a^{\sqrt{x+h}-\sqrt{x}}-1}{\sqrt{x+h}-\sqrt{x}} \times \lim\limits_{h \to 0} \frac{\sqrt{x+h}-\sqrt{x}}{h}$

Use the formula: $\lim\limits_{x \to 0} \frac{a^x-1}{x} = \log_e a$

$\Rightarrow f'(x) = a^{\sqrt{x}} \times \log_e a \times \lim\limits_{h \to 0} \frac{\sqrt{x+h}-\sqrt{x}}{h}$

Again we get an indeterminate form, so multiplying and dividing

$\sqrt{(x + h)} + \sqrt{(x)}$ to get rid of indeterminate form.

$\therefore f'(x) = a^{\sqrt{x}} \log_e a \lim\limits_{h \to 0} \frac{\sqrt{x+h}-\sqrt{x}}{h} \times \frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}$

Using $a^2 - b^2 = (a + b)(a - b)$, we have –

$\Rightarrow f'(x) = a^{\sqrt{x}} \log_e a \lim\limits_{h \to 0} \frac{(\sqrt{x+h})^2-(\sqrt{x})^2}{h(\sqrt{x+h}+\sqrt{x})}$

Using algebra of limits we have –

$\Rightarrow f'(x) = a^{\sqrt{x}} \log_e a \lim\limits_{h \to 0} \frac{x+h-x}{h} \times \lim\limits_{h \to 0} \frac{1}{(\sqrt{x+h}+\sqrt{x})}$

$\Rightarrow f'(x) = a^{\sqrt{x}} \log_e a \lim\limits_{h \to 0} \frac{h}{h} \times \frac{1}{\sqrt{x+(0)}+\sqrt{x}}$

$\Rightarrow f'(x) = a^{\sqrt{x}} \log_e a \lim\limits_{h \to 0} 1 \times \frac{1}{2\sqrt{x}}$

$\therefore f'(x) = \frac{a^{\sqrt{x}}}{2\sqrt{x}} \log_e a$

Hence,

Derivative of $f(x) = a^{\sqrt{x}} = \frac{a^{\sqrt{x}}}{2\sqrt{x}} \log_e a$

3 L. Question

Differentiate the following from first principles

3^{x^2}

Answer

We need to find derivative of $f(x) = 3^{x^2}$

Derivative of a function f(x) is given by –

$f'(x) = \lim\limits_{h \to 0} \frac{f(x+h)-f(x)}{h}$ {where h is a very small positive number}

$\therefore$ derivative of $f(x) = 3^{x^2}$ is given as –

$f'(x) = \lim\limits_{h \to 0} \frac{f(x+h)-f(x)}{h}$

$\Rightarrow f'(x) = \lim\limits_{h \to 0} \frac{3^{(x+h)^2}-3^{x^2}}{h}$

$\Rightarrow f'(x) = \lim\limits_{h \to 0} \frac{3^{x^2+2hx+h^2}-3^{x^2}}{h}$

Taking 3^{x^2} common, we have –

$\Rightarrow f'(x) = \lim_{h \to 0} \frac{3^{x^2}(3^{2hx+h^2}-1)}{h}$

Using algebra of limits –

$\Rightarrow f'(x) = \lim_{h \to 0} 3^{x^2} \times \lim_{h \to 0} \frac{3^{2hx+h^2}-1}{h}$

$\Rightarrow f'(x) = 3^{x^2} \lim_{h \to 0} \frac{3^{2hx+h^2}-1}{h}$

As one of the limits $\lim_{h \to 0} \frac{3^{2hx+h^2}-1}{h}$ can't be evaluated by directly putting the value of h as it will take 0/0 form.

So we need to take steps to find its value.

As h → 0 so, $(2hx + h^2) \to 0$

∴ multiplying numerator and denominator by $(2hx + h^2)$ in order to apply the formula – $\lim_{x \to 0} \frac{a^x-1}{x} = \log_e a$

$\therefore f'(x) = 3^{x^2} \lim_{h \to 0} \frac{3^{2hx+h^2}-1}{2hx+h^2} \times \frac{2hx+h^2}{h}$

Again using algebra of limits, we have –

$\Rightarrow f'(x) = 3^{x^2} \lim_{h \to 0} \frac{3^{2hx+h^2}-1}{2hx+h^2} \times \lim_{h \to 0} \frac{h(2x+h)}{h}$

Use the formula: $\lim_{x \to 0} \frac{a^x-1}{x} = \log_e a$

$\Rightarrow f'(x) = 3^{x^2} \times \log_e 3 \times \lim_{h \to 0} \frac{h(2x+h)}{h}$

$\Rightarrow f'(x) = 3^{x^2} \log_e 3 \lim_{h \to 0} (2x + h)$

$\therefore f'(x) = 3^{x^2} \log_e 3 \times (2x + 0) = 2x\, 3^{x^2} \log_e 3$

Hence,

Derivative of $f(x) = 3^{x^2} = 2x\, 3^{x^2} \log_e 3$

4 A. Question

Differentiate the following from first principles

$\tan^2 x$

Answer

We need to find derivative of $f(x) = \tan^2 x$

Derivative of a function f(x) is given by –

$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$ {where h is a very small positive number}

∴ derivative of $f(x) = \tan^2 x$ is given as –

$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$

$\Rightarrow f'(x) = \lim_{h \to 0} \frac{\tan^2(x+h)-\tan^2 x}{h}$

Using $(a + b)(a - b) = a^2 - b^2$,we have –

$\Rightarrow f'(x) = \lim_{h \to 0} \frac{\{\tan(x+h)-\tan x\}\{\tan(x+h)+\tan x\}}{h}$

Using algebra of limits, we have –

$\Rightarrow f'(x) = \lim_{h \to 0} \frac{\{\tan(x+h) - \tan x\}}{h} \times \lim_{h \to 0} \{\tan(x + h) + \tan x\}$

$\Rightarrow f'(x) = \{\tan(x + 0) + \tan x\} \times \lim_{h \to 0} \frac{\{\tan(x+h) - \tan x\}}{h}$

$\Rightarrow f'(x) = 2\tan x \lim_{h \to 0} \frac{\{\tan(x+h) - \tan x\}}{h}$

$\Rightarrow f'(x) = 2\tan x \lim_{h \to 0} \frac{\frac{\sin x + h}{\cos x + h} - \frac{\sin x}{\cos x}}{h}$

$\Rightarrow f'(x) = 2\tan x \lim_{h \to 0} \frac{\cos x \sin(x+h) - \sin x \cos(x+h)}{h\{\cos x \cos(x+h)\}}$

Using: sin A cos B – cos A sin B = sin (A – B)

$\Rightarrow f'(x) = 2\tan x \lim_{h \to 0} \frac{\sin(x+h-x)}{h\{\cos x \cos(x+h)\}}$

Using algebra of limits we have –

$\therefore f'(x) = 2\tan x \lim_{h \to 0} \frac{\sin(h)}{h} \times \lim_{h \to 0} \frac{1}{\{\cos x \cos(x+h)\}}$

Use the formula – $\lim_{x \to 0} \frac{\sin x}{x} = 1$

$\therefore f'(x) = 2\tan x \times 1 \times \frac{1}{\{\cos x \cos(x+0)\}}$

$\therefore f'(x) = \frac{2\tan x}{\cos^2 x} = 2\tan x \sec^2 x$

Hence,

Derivative of f(x) = ($\tan^2 x$) is ($2 \tan x \sec^2 x$)

4 B. Question

Differentiate the following from first principles

tan (2x + 1)

Answer

We need to find derivative of f(x) = tan (2x + 1)

Derivative of a function f(x) is given by –

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ {where h is a very small positive number}

∴ derivative of f(x) = tan (2x + 1) is given as –

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$\Rightarrow f'(x) = \lim_{h \to 0} \frac{\tan(2(x+h)+1) - \tan(2x+1)}{h}$

$\Rightarrow f'(x) = \lim_{h \to 0} \frac{\tan(2x+2h+1) - \tan(2x+1)}{h}$

$\Rightarrow f'(x) = \lim_{h \to 0} \frac{\frac{\sin(2x+2h+1)}{\cos(2x+2h+1)} - \frac{\sin(2x+1)}{\cos(2x+1)}}{h}$

$\Rightarrow f'(x) = \lim_{h \to 0} \frac{\cos(2x+1)\sin(2x+2h+1) - \sin(2x+1)\cos(2x+2h+1)}{h\{\cos(2x+1)\cos(2x+2h+1)\}}$

Using: sin A cos B – cos A sin B = sin (A – B)

$$\Rightarrow f'(x) = \lim_{h \to 0} \frac{\sin(2x + 2h + 1 - 2x - 1)}{h\{\cos(2x + 1)\cos(2x + 2h + 1)\}}$$

Using algebra of limits we have –

$$\therefore f'(x) = \lim_{h \to 0} \frac{\sin(2h)}{h} \times \lim_{h \to 0} \frac{1}{\{\cos(2x + 1)\cos(2x + 2h + 1)\}}$$

To apply sandwich theorem ,we need 2h in denominator, So multiplying by 2 in numerator and denominator by 2.

$$\therefore f'(x) = 2\lim_{h \to 0} \frac{\sin(2h)}{2h} \times \lim_{h \to 0} \frac{1}{\{\cos(2x + 1)\cos(2x + 2h + 1)\}}$$

Use the formula – $\lim_{x \to 0} \frac{\sin x}{x} = 1$

$$\Rightarrow f'(x) = 2 \times \frac{1}{\cos^2(2x + 1)}$$

$$\therefore f'(x) = 2\sec^2(2x + 1)$$

Hence,

Derivative of f(x) = tan(2x + 1) is 2 $\sec^2$ (2x + 1)

4 C. Question

Differentiate the following from first principles

tan 2x

Answer

We need to find derivative of f(x) = tan (2x)

Derivative of a function f(x) is given by –

$f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}$ {where h is a very small positive number}

∴ derivative of f(x) = tan (2x) is given as –

$$f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}$$

$$\Rightarrow f'(x) = \lim_{h \to 0} \frac{\tan(2(x + h)) - \tan(2x)}{h}$$

$$\Rightarrow f'(x) = \lim_{h \to 0} \frac{\tan(2x + 2h) - \tan(2x)}{h}$$

$$\Rightarrow f'(x) = \lim_{h \to 0} \frac{\frac{\sin(2x + 2h)}{\cos(2x + 2h)} - \frac{\sin(2x)}{\cos(2x)}}{h}$$

$$\Rightarrow f'(x) = \lim_{h \to 0} \frac{\cos(2x)\sin(2x + 2h) - \sin(2x)\cos(2x + 2h)}{h\{\cos(2x)\cos(2x + 2h)\}}$$

Using: sin A cos B – cos A sin B = sin (A – B)

$$\Rightarrow f'(x) = \lim_{h \to 0} \frac{\sin(2x + 2h - 2x)}{h\{\cos(2x)\cos(2x + 2h)\}}$$

Using algebra of limits we have –

$$\therefore f'(x) = \lim_{h \to 0} \frac{\sin(2h)}{h} \times \lim_{h \to 0} \frac{1}{\{\cos(2x)\cos(2x + 2h)\}}$$

To apply sandwich theorem ,we need 2h in denominator, So multiplying by 2 in numerator and denominator by 2.

$$\therefore f'(x) = 2\lim_{h \to 0} \frac{\sin(2h)}{2h} \times \lim_{h \to 0} \frac{1}{\{\cos(2x)\cos(2x + 2h)\}}$$

Use the formula – $\lim_{x \to 0} \frac{\sin x}{x} = 1$

$\Rightarrow$ f'(x) = 2× $\frac{1}{\cos^2(2x)}$

∴ f'(x) = $2\sec^2(2x)$

Hence,

Derivative of f(x) = tan(2x) is 2 $\sec^2$ (2x)

4 D. Question

Differentiate the following from first principles

$\sqrt{\tan x}$

Answer

We need to find derivative of f(x) = √tan x

Derivative of a function f(x) is given by –

f'(x) = $\lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$ {where h is a very small positive number}

∴ derivative of f(x) = √tan x is given as –

f'(x) = $\lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$

$\Rightarrow$ f'(x) = $\lim_{h \to 0} \frac{\sqrt{\tan(x+h)}-\sqrt{\tan x}}{h}$

As the limit takes 0/0 form on putting h = 0. So we need to remove the indeterminate form. As the numerator expression has square root terms so we need to multiply numerator and denominator by √tan (x + h) + √tan x.

$\Rightarrow$ f'(x) = $\lim_{h \to 0} \frac{\sqrt{\tan(x+h)}-\sqrt{\tan x}}{h} \times \lim_{h \to 0} \frac{\sqrt{\tan(x+h)}+\sqrt{\tan x}}{\sqrt{\tan(x+h)}+\sqrt{\tan x}}$

Using (a + b)(a – b) = $a^2 - b^2$,we have –

$\Rightarrow$ f'(x) = $\lim_{h \to 0} \frac{\left(\sqrt{\tan(x+h)}\right)^2-\left(\sqrt{\tan x}\right)^2}{h\left(\sqrt{\tan(x+h)}+\sqrt{\tan x}\right)}$

Using algebra of limits, we have –

$\Rightarrow$ f'(x) = $\lim_{h \to 0} \frac{\tan(x+h)-\tan x}{h} \times \lim_{h \to 0} \frac{1}{\sqrt{\tan(x+h)}+\sqrt{\tan x}}$

$\Rightarrow$ f'(x) = $\frac{1}{\sqrt{\tan(x+0)}+\sqrt{\tan x}} \times \lim_{h \to 0} \frac{\tan(x+h)-\tan x}{h}$

$\Rightarrow$ f'(x) = $\frac{1}{2\sqrt{\tan x}} \lim_{h \to 0} \frac{\frac{\sin(x+h)}{\cos(x+h)}-\frac{\sin(x)}{\cos(x)}}{h}$

$\Rightarrow$ f'(x) = $\frac{1}{2\sqrt{\tan x}} \lim_{h \to 0} \frac{\cos(x)\sin(x+h)-\sin(x)\cos(x+h)}{h\{\cos(x)\cos(x+h)\}}$

Using: sin A cos B – cos A sin B = sin (A – B)

$\Rightarrow$ f'(x) = $\frac{1}{2\sqrt{\tan x}} \lim_{h \to 0} \frac{\sin(x+h-x)}{h\{\cos(x)\cos(x+h)\}}$

Using algebra of limits we have –

∴ f'(x) = $\frac{1}{2\sqrt{\tan x}} \lim_{h \to 0} \frac{\sin(h)}{h} \times \lim_{h \to 0} \frac{1}{\{\cos(x)\cos(x+h)\}}$

Use the formula – $\lim\limits_{x \to 0} \frac{\sin x}{x} = 1$

$\Rightarrow$ f'(x) = $\frac{1}{2\sqrt{\tan x}} \times \frac{1}{\cos^2(x)}$

$\therefore$ f'(x) = $\frac{\sec^2 x}{2\sqrt{\tan x}}$

Hence,

Derivative of f(x) = √tan(x) is $\frac{\sec^2 x}{2\sqrt{\tan x}}$

5 A. Question

Differentiate the following from first principles

$\sin\sqrt{2x}$

Answer

We need to find derivative of f(x) = $\sin\sqrt{2x}$

Derivative of a function f(x) from first principle is given by –

f'(x) = $\lim\limits_{h \to 0} \frac{f(x+h)-f(x)}{h}$ {where h is a very small positive number}

$\therefore$ derivative of f(x) = $\sin\sqrt{2x}$ is given as –

f'(x) = $\lim\limits_{h \to 0} \frac{f(x+h)-f(x)}{h}$

$\Rightarrow$ f'(x) = $\lim\limits_{h \to 0} \frac{\sin\sqrt{2(x+h)}-\sin\sqrt{2x}}{h}$

Use: sin A – sin B = 2 cos ((A + B)/2) sin ((A – B)/2)

$\Rightarrow$ f'(x) = $\lim\limits_{h \to 0} \frac{2\cos\left(\frac{\sqrt{2x+2h}+\sqrt{2x}}{2}\right)\sin\left(\frac{\sqrt{2x+2h}-\sqrt{2x}}{2}\right)}{h}$

Using algebra of limits, we have –

$\Rightarrow$ f'(x) = 2$\lim\limits_{h \to 0} \cos\left(\frac{\sqrt{2x+2h}+\sqrt{2x}}{2}\right) \times \lim\limits_{h \to 0} \frac{\sin\frac{\sqrt{2x+2h}-\sqrt{2x}}{2}}{h}$

$\Rightarrow$ f'(x) = 2 $\cos\left(\frac{\sqrt{2x+2(0)}+\sqrt{2x}}{2}\right) \times \lim\limits_{h \to 0} \frac{\sin\left(\frac{\sqrt{2x+2h}-\sqrt{2x}}{2}\right)}{h}$

$\Rightarrow$ f'(x) = $2\cos\sqrt{2x} \times \lim\limits_{h \to 0} \frac{\sin\left(\frac{\sqrt{2x+2h}-\sqrt{2x}}{2}\right)}{h}$

As, h → 0 ⇒ $\sqrt{2x+2h} - \sqrt{2x}$ → 0

$\therefore$ To use the sandwich theorem to evaluate the limit, we need $\frac{\sqrt{2x+2h}-\sqrt{2x}}{2}$ in denominator. So multiplying this in numerator and denominator.

$\Rightarrow$ f'(x) = $2\cos\sqrt{2x} \times \lim\limits_{h \to 0} \frac{\sin\left(\frac{\sqrt{2x+2h}-\sqrt{2x}}{2}\right)}{h\left(\frac{\sqrt{2x+2h}-\sqrt{2x}}{2}\right)} \times \left(\frac{\sqrt{2x+2h}-\sqrt{2x}}{2}\right)$

Using algebra of limits –

$\Rightarrow$ f'(x) = $2\cos\sqrt{2x} \times \lim\limits_{h \to 0} \frac{\sin\left(\frac{\sqrt{2x+2h}-\sqrt{2x}}{2}\right)}{\left(\frac{\sqrt{2x+2h}-\sqrt{2x}}{2}\right)} \times \lim\limits_{h \to 0} \frac{\sqrt{2x+2h}-\sqrt{2x}}{2h}$

Use the formula: $\lim_{x \to 0} \frac{\sin x}{x} = 1$

$\therefore f'(x) = 2\cos\sqrt{2x} \times 1 \times \lim_{h \to 0} \frac{\sqrt{2x+2h}-\sqrt{2x}}{2h}$

$\Rightarrow f'(x) = 2\cos\sqrt{2x} \lim_{h \to 0} \frac{\sqrt{2x+2h}-\sqrt{2x}}{2h}$

$\Rightarrow f'(x) = 2\cos\sqrt{2x} \lim_{h \to 0} \frac{\sqrt{2x+2h}-\sqrt{2x}}{2h}$

Again we get an indeterminate form, so multiplying and dividing √(2x + 2h) + √(2x) to get rid of indeterminate form.

$\therefore f'(x) = 2\cos\sqrt{2x} \lim_{h \to 0} \frac{\sqrt{2x+2h}-\sqrt{2x}}{2h} \times \frac{\sqrt{2x+2h}+\sqrt{2x}}{\sqrt{2x+2h}+\sqrt{2x}}$

Using $a^2 - b^2 = (a + b)(a - b)$, we have –

$\Rightarrow f'(x) = 2\cos\sqrt{2x} \lim_{h \to 0} \frac{\left(\sqrt{2x+2h}\right)^2-\left(\sqrt{2x}\right)^2}{2h\left(\sqrt{2x+2h}+\sqrt{2x}\right)}$

Using algebra of limits we have –

$\Rightarrow f'(x) = 2\cos\sqrt{2x} \lim_{h \to 0} \frac{2x+2h-2h}{2h} \times \lim_{h \to 0} \frac{1}{\sqrt{2x+2h}+\sqrt{2x}}$

$\Rightarrow f'(x) = 2\cos\sqrt{2x} \lim_{h \to 0} \frac{2h}{2h} \times \frac{1}{\sqrt{2x+2(0)}+\sqrt{2x}}$

$\Rightarrow f'(x) = 2\cos\sqrt{2x} \times \lim_{h \to 0} 1 \times \frac{1}{2\sqrt{2x}}$

$\therefore f'(x) = \frac{\cos\sqrt{2x}}{\sqrt{2x}}$

Hence,

Derivative of f(x) = sin √2x = $\frac{\cos\sqrt{2x}}{\sqrt{2x}}$

5 B. Question

Differentiate the following from first principles

$\cos\sqrt{x}$

Answer

We need to find derivative of f(x) = $\cos\sqrt{x}$

Derivative of a function f(x) from first principle is given by –

$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$ {where h is a very small positive number}

∴ derivative of f(x) = $\cos\sqrt{x}$ is given as –

$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$

$\Rightarrow f'(x) = \lim_{h \to 0} \frac{\cos\sqrt{(x+h)}-\cos\sqrt{x}}{h}$

Use: cos A – cos B = – 2 sin ((A + B)/2) sin ((A – B)/2)

$\Rightarrow f'(x) = \lim_{h \to 0} \frac{-2\sin\left(\frac{\sqrt{x+h}+\sqrt{x}}{2}\right)\sin\left(\frac{\sqrt{x+h}-\sqrt{x}}{2}\right)}{h}$

Using algebra of limits, we have –

$\Rightarrow$ f'(x) = - 2 $\lim_{h\to 0} \sin\left(\frac{\sqrt{x+h}+\sqrt{x}}{2}\right) \times \lim_{h\to 0} \frac{\sin\frac{\sqrt{x+h}-\sqrt{x}}{2}}{h}$

$\Rightarrow$ f'(x) = - 2 $\sin\left(\frac{\sqrt{x+0}+\sqrt{x}}{2}\right) \times \lim_{h\to 0} \frac{\sin\left(\frac{\sqrt{x+h}-\sqrt{x}}{2}\right)}{h}$

$\Rightarrow$ f'(x) = $-2\sin\sqrt{x} \times \lim_{h\to 0} \frac{\sin\left(\frac{\sqrt{x+h}-\sqrt{x}}{2}\right)}{h}$

As, h → 0 ⇒ $\sqrt{x+h} - \sqrt{x} \to 0$

∴ To use the sandwich theorem to evaluate the limit, we need $\frac{\sqrt{x+h}-\sqrt{x}}{2}$ in denominator. So multiplying this in numerator and denominator.

$\Rightarrow$ f'(x) = $-2\sin\sqrt{x} \times \lim_{h\to 0} \frac{\sin\left(\frac{\sqrt{x+h}-\sqrt{x}}{2}\right)}{h\left(\frac{\sqrt{x+h}-\sqrt{x}}{2}\right)} \times \left(\frac{\sqrt{x+h}-\sqrt{x}}{2}\right)$

Using algebra of limits –

$\Rightarrow$ f'(x) = $-2\sin\sqrt{x} \times \lim_{h\to 0} \frac{\sin\left(\frac{\sqrt{x+h}-\sqrt{x}}{2}\right)}{\left(\frac{\sqrt{x+h}-\sqrt{x}}{2}\right)} \times \lim_{h\to 0} \frac{\sqrt{x+h}-\sqrt{x}}{2h}$

Use the formula: $\lim_{x\to 0} \frac{\sin x}{x} = 1$

∴ f'(x) = $-2\sin\sqrt{x} \times 1 \times \lim_{h\to 0} \frac{\sqrt{x+h}-\sqrt{x}}{2h}$

$\Rightarrow$ f'(x) = $-2\sin\sqrt{x}\ \lim_{h\to 0} \frac{\sqrt{x+h}-\sqrt{x}}{2h}$

Again, we get an indeterminate form, so multiplying and dividing √(x + h) + √(x) to get rid of indeterminate form.

∴ f'(x) = $-2\sin\sqrt{x}\ \lim_{h\to 0} \frac{\sqrt{x+h}-\sqrt{x}}{2h} \times \frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}$

Using $a^2 - b^2 = (a + b)(a - b)$, we have –

$\Rightarrow$ f'(x) = $-2\sin\sqrt{x}\ \lim_{h\to 0} \frac{(\sqrt{x+h})^2-(\sqrt{x})^2}{2h(\sqrt{x+h}+\sqrt{x})}$

Using algebra of limits we have –

$\Rightarrow$ f'(x) = $-2\sin\sqrt{x}\ \lim_{h\to 0} \frac{x+h-x}{2h} \times \lim_{h\to 0} \frac{1}{\sqrt{x+h}+\sqrt{x}}$

$\Rightarrow$ f'(x) = $-2\sin\sqrt{x}\ \lim_{h\to 0} \frac{h}{2h} \times \frac{1}{\sqrt{x+(0)}+\sqrt{x}}$

$\Rightarrow$ f'(x) = $-2\sin\sqrt{x} \times \lim_{h\to 0} \frac{1}{2} \times \frac{1}{2\sqrt{x}}$

∴ f'(x) = $\frac{-\sin\sqrt{x}}{2\sqrt{x}}$

Hence,

Derivative of f(x) = cos √x = $\frac{-\sin\sqrt{x}}{2\sqrt{x}}$

5 C. Question

Differentiate the following from first principles

$\tan\sqrt{x}$

Answer

We need to find derivative of f(x) = $\tan\sqrt{x}$

Derivative of a function f(x) from first principle is given by –

f'(x) = $\lim\limits_{h\to 0}\frac{f(x+h)-f(x)}{h}$ {where h is a very small positive number}

∴ derivative of f(x) = $\tan\sqrt{x}$ is given as –

f'(x) = $\lim\limits_{h\to 0}\frac{f(x+h)-f(x)}{h}$

⇒ f'(x) = $\lim\limits_{h\to 0}\frac{\tan\sqrt{(x+h)}-\tan\sqrt{x}}{h}$

⇒ f'(x) = $\lim\limits_{h\to 0}\frac{\frac{\sin\sqrt{(x+h)}}{\cos\sqrt{x+h}}-\frac{\sin\sqrt{x}}{\cos\sqrt{x}}}{h}$

⇒ f'(x) = $\lim\limits_{h\to 0}\frac{\cos\sqrt{x}\sin\sqrt{x+h}-\cos\sqrt{x+h}\sin\sqrt{x}}{h\cos\sqrt{x}\cos(\sqrt{x+h})}$

Use the formula: sin (A – B) = sin A cos B – cos A sin B

⇒ f'(x) = $\lim\limits_{h\to 0}\frac{\sin\{\sqrt{x+h}-\sqrt{x}\}}{h\cos\sqrt{x}\cos(\sqrt{x+h})}$

Using algebra of limits, we have –

⇒ f'(x) = $\lim\limits_{h\to 0}\frac{\sin(\sqrt{x+h}-\sqrt{x})}{h}\times\lim\limits_{h\to 0}\frac{1}{\cos\sqrt{x}\cos(\sqrt{x+h})}$

⇒ f'(x) = $\frac{1}{\cos\sqrt{x}\cos\sqrt{x+0}}\times\lim\limits_{h\to 0}\frac{\sin(\sqrt{x+h}-\sqrt{x})}{h}$

⇒ f'(x) = $\frac{1}{\cos^2\sqrt{x}}\times\lim\limits_{h\to 0}\frac{\sin(\sqrt{x+h}-\sqrt{x})}{h}$

⇒ f'(x) = $\frac{1}{\cos^2\sqrt{x}}\times\lim\limits_{h\to 0}\frac{\sin(\sqrt{x+h}-\sqrt{x})}{h}$

As, h → 0 ⇒ $\sqrt{x+h}-\sqrt{x}$ → 0

∴ To use the sandwich theorem to evaluate the limit, we need $\sqrt{x+h}-\sqrt{x}$ in denominator. So multiplying this in numerator and denominator.

⇒ f'(x) = $\frac{1}{\cos^2\sqrt{x}}\times\lim\limits_{h\to 0}\frac{\sin(\sqrt{x+h}-\sqrt{x})}{h(\sqrt{x+h}-\sqrt{x})}\times(\sqrt{x+h}-\sqrt{x})$

Using algebra of limits –

⇒ f'(x) = $\frac{1}{\cos^2\sqrt{x}}\times\lim\limits_{h\to 0}\frac{\sin(\sqrt{x+h}-\sqrt{x})}{(\sqrt{x+h}-\sqrt{x})}\times\lim\limits_{h\to 0}\frac{\sqrt{x+h}-\sqrt{x}}{h}$

Use the formula: $\lim\limits_{x\to 0}\frac{\sin x}{x} = 1$

∴ f'(x) = $\frac{1}{\cos^2\sqrt{x}}\times 1\times\lim\limits_{h\to 0}\frac{\sqrt{x+h}-\sqrt{x}}{h}$

⇒ f'(x) = $\frac{1}{\cos^2\sqrt{x}}\lim\limits_{h\to 0}\frac{\sqrt{x+h}-\sqrt{x}}{h}$

Again, we get an indeterminate form, so multiplying and dividing √(x + h) + √(x) to get rid of indeterminate form.

∴ f'(x) = $\frac{1}{\cos^2\sqrt{x}}\lim\limits_{h\to 0}\frac{\sqrt{x+h}-\sqrt{x}}{h}\times\frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}$

Using $a^2 - b^2 = (a + b)(a - b)$, we have –

$$\Rightarrow f'(x) = \frac{1}{\cos^2\sqrt{x}} \lim_{h \to 0} \frac{\left(\sqrt{x+h}\right)^2 - \left(\sqrt{x}\right)^2}{h\left(\sqrt{x+h}+\sqrt{x}\right)}$$

Using algebra of limits we have –

$$\Rightarrow f'(x) = \frac{1}{\cos^2\sqrt{x}} \lim_{h \to 0} \frac{x+h-x}{h} \times \lim_{h \to 0} \frac{1}{\sqrt{x+h}+\sqrt{x}}$$

$$\Rightarrow f'(x) = \frac{1}{\cos^2\sqrt{x}} \lim_{h \to 0} \frac{h}{h} \times \frac{1}{\sqrt{x+(0)}+\sqrt{x}}$$

$$\Rightarrow f'(x) = \frac{1}{\cos^2\sqrt{x}} \times \lim_{h \to 0} 1 \times \frac{1}{2\sqrt{x}}$$

$$\therefore f'(x) = \frac{\sec^2\sqrt{x}}{2\sqrt{x}}$$

Hence,

Derivative of $f(x) = \tan\sqrt{x} = \frac{\sec^2\sqrt{x}}{2\sqrt{x}}$

5 D. Question

Differentiate the following from first principles

$\tan x^2$

Answer

We need to find derivative of $f(x) = \tan x^2$

Derivative of a function f(x) from first principle is given by –

$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$ {where h is a very small positive number}

$\therefore$ derivative of $f(x) = \tan x^2$ is given as –

$$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$$

$$\Rightarrow f'(x) = \lim_{h \to 0} \frac{\tan(x+h)^2 - \tan x^2}{h}$$

$$\Rightarrow f'(x) = \lim_{h \to 0} \frac{\frac{\sin(x+h)^2}{\cos(x+h)^2} - \frac{\sin x^2}{\cos x^2}}{h}$$

$$\Rightarrow f'(x) = \lim_{h \to 0} \frac{\cos x^2 \sin(x+h)^2 - \cos(x+h)^2 \sin x^2}{h \cos x^2 \cos(x+h)^2}$$

Use the formula: sin (A – B) = sin A cos B – cos A sin B

$$\Rightarrow f'(x) = \lim_{h \to 0} \frac{\sin\{(x+h)^2 - x^2\}}{h \cos x^2 \cos(x+h)^2}$$

Using algebra of limits, we have –

$$\Rightarrow f'(x) = \lim_{h \to 0} \frac{\sin\left(x^2 + 2hx + h^2 - x^2\right)}{h} \times \lim_{h \to 0} \frac{1}{\cos x^2 \cos(x+h)^2}$$

$$\Rightarrow f'(x) = \frac{1}{\cos x^2 \cos(x+0)^2} \times \lim_{h \to 0} \frac{\sin\left(2hx + h^2\right)}{h}$$

$$\Rightarrow f'(x) = \frac{1}{\cos^2(x^2)} \times \lim_{h \to 0} \frac{\sin\left(2hx + h^2\right)}{h}$$

As, $h \to 0 \Rightarrow 2hx + h^2 \to 0$

∴ To use the sandwich theorem to evaluate the limit, we need $2hx + h^2$ in denominator. So multiplying this in numerator and denominator.

$$\Rightarrow f'(x) = \sec^2 x^2 \times \lim_{h \to 0} \frac{\sin(2hx + h^2)}{h(2hx + h^2)} \times (2hx + h^2)$$

Using algebra of limits –

$$\Rightarrow f'(x) = \sec^2 x^2 \times \lim_{h \to 0} \frac{\sin(2hx + h^2)}{(2hx + h^2)} \times \lim_{h \to 0} \frac{h(2x + h)}{h}$$

$$\Rightarrow f'(x) = \sec^2 x^2 \times \lim_{h \to 0} \frac{\sin(2hx + h^2)}{(2hx + h^2)} \times \lim_{h \to 0} 2x + h$$

Use the formula: $\lim_{x \to 0} \frac{\sin x}{x} = 1$

∴ $f'(x) = \sec^2 x^2 \times 1 \times (2x + 0)$

∴ $f'(x) = 2x \sec^2 x^2$

Hence,

Derivative of $f(x) = \tan x^2 = 2x \sec^2 x^2$

Exercise 30.3

1. Question

Differentiate the following with respect to x:

$x^4 - 2\sin x + 3 \cos x$

Answer

Given,

$f(x) = x^4 - 2\sin x + 3 \cos x$

we need to find f'(x), so differentiating both sides with respect to x –

$$\therefore \frac{d}{dx}\{f(x)\} = \frac{d}{dx}(x^4 - 2\sin x + 3\cos x)$$

Using algebra of derivatives –

$$\Rightarrow f'(x) = \frac{d}{dx}(x^4) - 2\frac{d}{dx}(\sin x) + 3\frac{d}{dx}(\cos x)$$

Use the formula: $\frac{d}{dx}(x^n) = nx^{n-1}, \frac{d}{dx}(\sin x) = \cos x, \frac{d}{dx}(\cos x) = -\sin x$

∴ $f'(x) = 4x^{4-1} - 2 \cos x + 3(-\sin x)$

⇒ $f'(x) = 4x^3 - 2 \cos x - 3 \sin x$

2. Question

Differentiate the following with respect to x:

$3^x + x^3 + 3^3$

Answer

Given,

$f(x) = 3^x + x^3 + 3^3$

we need to find f'(x), so differentiating both sides with respect to x –

$$\therefore \frac{d}{dx}\{f(x)\} = \frac{d}{dx}(3^x + x^3 + 3^3)$$

Using algebra of derivatives –

$\Rightarrow f'(x) = \frac{d}{dx}(3^x) + \frac{d}{dx}(x^3) + \frac{d}{dx}(3^3)$

Use the formula: $\frac{d}{dx}(x^n) = nx^{n-1}, \frac{d}{dx}(a^x) = a^x \log a, \frac{d}{dx}(\text{constant}) = 0$

$\therefore f'(x) = 3^x \log_e 3 - 3x^{3-1} + 0$

$\Rightarrow f'(x) = 3^x \log_e 3 - 3x^2$

3. Question

Differentiate the following with respect to x:

$$\frac{x^3}{3} - 2\sqrt{x} + \frac{5}{x^2}$$

Answer

Given,

$f(x) = \frac{x^3}{3} - 2\sqrt{x} + \frac{5}{x^2}$

we need to find f'(x), so differentiating both sides with respect to x –

$\therefore \frac{d}{dx}\{f(x)\} = \frac{d}{dx}\left(\frac{x^3}{3} - 2\sqrt{x} + \frac{5}{x^2}\right)$

Using algebra of derivatives –

$\Rightarrow f'(x) = \frac{d}{dx}\left(\frac{x^3}{3}\right) - 2\frac{d}{dx}(\sqrt{x}) + 5\frac{d}{dx}\left(\frac{1}{x^2}\right)$

$\Rightarrow f'(x) = \frac{1}{3}\frac{d}{dx}(x^3) - 2\frac{d}{dx}(x^{\frac{1}{2}}) + 5\frac{d}{dx}(x^{-2})$

Use the formula: $\frac{d}{dx}(x^n) = nx^{n-1}$

$\therefore f'(x) = \frac{1}{3}(3x^{3-1}) - 2 \times \frac{1}{2} x^{\frac{1}{2}-1} + 5(-2)x^{-2-1}$

$\Rightarrow f'(x) = 3 \times \frac{1}{3} x^2 - x^{-\frac{1}{2}} - 10x^{-3}$

$\therefore f'(x) = x^2 - x^{(-1/2)} - 10x^{-3}$

4. Question

Differentiate the following with respect to x:

$e^{x \log a} + e^{a \log x} + e^{a \log a}$

Answer

Given,

$f(x) = e^{x \log a} + e^{a \log x} + e^{a \log a}$

$\Rightarrow f(x) = e^{\log a^x} + e^{\log_e x^a} + e^{\log_e a^a}$

We know that, $e^{\log f(x)} = f(x)$

$\therefore f(x) = a^x + x^a + a^a$

we need to find f'(x), so differentiating both sides with respect to x –

$\therefore \frac{d}{dx}\{f(x)\} = \frac{d}{dx}(a^x + x^a + a^a)$

Using algebra of derivatives –

$\Rightarrow f'(x) = \frac{d}{dx}(a^x) + \frac{d}{dx}(x^a) + \frac{d}{dx}(a^a)$

Use the formula: $\frac{d}{dx}(x^n) = nx^{n-1}, \frac{d}{dx}(a^x) = a^x \log a, \frac{d}{dx}(\text{constant}) = 0$

$\therefore f'(x) = a^x \log_e a - ax^{a-1} + 0$

$\Rightarrow f'(x) = a^x \log_e a - ax^{a-1}$

5. Question

Differentiate the following with respect to x:

$(2x^2 + 1)(3x + 2)$

Answer

Given,

$f(x) = (2x^2 + 1)(3x + 2)$

$\Rightarrow f(x) = 6x^3 + 4x^2 + 3x + 2$

we need to find f′(x), so differentiating both sides with respect to x –

$\therefore \frac{d}{dx}\{f(x)\} = \frac{d}{dx}(6x^3 + 4x^2 + 3x + 2)$

Using algebra of derivatives –

$\Rightarrow f'(x) = 6\frac{d}{dx}(x^3) + 4\frac{d}{dx}(x^2) + 3\frac{d}{dx}(x) + \frac{d}{dx}(2)$

Use the formula: $\frac{d}{dx}(x^n) = nx^{n-1}$ and $\frac{d}{dx}(\text{constant}) = 0$

$\therefore f'(x) = 6(3x^{3-1}) + 4(2x^{2-1}) + 3(x^{1-1}) + 0$

$\Rightarrow f'(x) = 18x^2 + 8x + 3 + 0$

$\therefore f'(x) = 18x^2 + 8x + 3$

6. Question

Differentiate the following with respect to x:

$\log_3 x + 3\log_e x + 2 \tan x$

Answer

Given,

$f(x) = \log_3 x + 3\log_e x + 2 \tan x$

we need to find f′(x), so differentiating both sides with respect to x –

$\therefore \frac{d}{dx}\{f(x)\} = \frac{d}{dx}(\log_3 x + 3\log_e x + 2\tan x)$

Using algebra of derivatives –

$\Rightarrow f'(x) = \frac{d}{dx}(\log_3 x) + 3\frac{d}{dx}(\log_e x) + 2\frac{d}{dx}(\tan x)$

Use the formula: $\frac{d}{dx}(\log_a x) = \frac{1}{x \log_e a}$ and $\frac{d}{dx}(\tan x) = \sec^2 x$

$\therefore f'(x) = \frac{1}{x \log_e 3} + \frac{3}{x \log_e e} + 2\sec^2 x$

$\Rightarrow f'(x) = \frac{1}{x \log_e 3} + \frac{3}{x} + 2\sec^2 x$

$\therefore f'(x) = \frac{1}{x \log_e 3} + \frac{3}{x} + 2\sec^2 x$

7. Question

Differentiate the following with respect to x:

$$\left(x + \frac{1}{x}\right)\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)$$

Answer

Given,

$f(x) = \left(x + \frac{1}{x}\right)\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)$

$\Rightarrow f(x) = x\sqrt{x} + \frac{x}{\sqrt{x}} + \frac{\sqrt{x}}{x} + \frac{1}{x\sqrt{x}}$

$\Rightarrow f(x) = x^{3/2} + x^{1/2} + x^{-1/2} + x^{-3/2}$

we need to find f'(x), so differentiating both sides with respect to x –

$\therefore \frac{d}{dx}\{f(x)\} = \frac{d}{dx}\left(x^{\frac{3}{2}} + x^{\frac{1}{2}} + x^{-\frac{1}{2}} + x^{-\frac{3}{2}}\right)$

Using algebra of derivatives –

$\Rightarrow f'(x) = \frac{d}{dx}\left(x^{\frac{3}{2}}\right) + \frac{d}{dx}\left(x^{\frac{1}{2}}\right) + \frac{d}{dx}\left(x^{-\frac{1}{2}}\right) + \frac{d}{dx}\left(x^{-3/2}\right)$

Use the formula: $\frac{d}{dx}(x^n) = nx^{n-1}$ and $\frac{d}{dx}(\text{constant}) = 0$

$\therefore f'(x) = \frac{3}{2}x^{\frac{3}{2}-1} + \frac{1}{2}x^{\frac{1}{2}-1} + \left(-\frac{1}{2}\right)x^{-\frac{1}{2}-1} + \left(-\frac{3}{2}\right)x^{-\frac{3}{2}-1}$

$\Rightarrow f'(x) = \frac{3}{2}x^{\frac{1}{2}} + \frac{1}{2}x^{-\frac{1}{2}} - \frac{1}{2}x^{-\frac{3}{2}} - \frac{3}{2}x^{-\frac{5}{2}}$

$\Rightarrow f'(x) = \frac{3}{2}\sqrt{x} + \frac{1}{2\sqrt{x}} - \frac{1}{2x\sqrt{x}} - \frac{3}{2x^2\sqrt{x}}$

$\therefore f'(x) = \frac{3}{2}\sqrt{x} + \frac{1}{2\sqrt{x}} - \frac{1}{2x\sqrt{x}} - \frac{3}{2x^2\sqrt{x}}$

8. Question

Differentiate the following with respect to x:

$$\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^3$$

Answer

Given,

$f(x) = \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^3$

Using $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$\Rightarrow f(x) = \left(\sqrt{x}\right)^3 + \frac{3\left(\sqrt{x}\right)^2}{\sqrt{x}} + \frac{3\sqrt{x}}{\left(\sqrt{x}\right)^2} + \frac{1}{\left(\sqrt{x}\right)^3}$

$\Rightarrow f(x) = x^{3/2} + x^{1/2} + x^{-1/2} + x^{-3/2}$

we need to find f′(x), so differentiating both sides with respect to x –

$\therefore \frac{d}{dx}\{f(x)\} = \frac{d}{dx}\left(x^{\frac{3}{2}} + 3x^{\frac{1}{2}} + 3x^{-\frac{1}{2}} + x^{-\frac{3}{2}}\right)$

Using algebra of derivatives –

$\Rightarrow f'(x) = \frac{d}{dx}\left(x^{\frac{3}{2}}\right) + 3\frac{d}{dx}\left(x^{\frac{1}{2}}\right) + 3\frac{d}{dx}\left(x^{-\frac{1}{2}}\right) + \frac{d}{dx}\left(x^{-3/2}\right)$

Use the formula: $\frac{d}{dx}(x^n) = nx^{n-1}$ and $\frac{d}{dx}(\text{constant}) = 0$

$\therefore f'(x) = \frac{3}{2}x^{\frac{3}{2}-1} + \frac{3}{2}x^{\frac{1}{2}-1} + \left(-\frac{3}{2}\right)x^{-\frac{1}{2}-1} + \left(-\frac{3}{2}\right)x^{-\frac{3}{2}-1}$

$\Rightarrow f'(x) = \frac{3}{2}x^{\frac{1}{2}} + \frac{3}{2}x^{-\frac{1}{2}} - \frac{3}{2}x^{-\frac{3}{2}} - \frac{3}{2}x^{-\frac{5}{2}}$

$\Rightarrow f'(x) = \frac{3}{2}\sqrt{x} + \frac{3}{2\sqrt{x}} - \frac{3}{2x\sqrt{x}} - \frac{3}{2x^2\sqrt{x}}$

$\therefore f'(x) = \frac{3}{2}\sqrt{x} + \frac{3}{2\sqrt{x}} - \frac{3}{2x\sqrt{x}} - \frac{3}{2x^2\sqrt{x}}$

9. Question

Differentiate the following with respect to x:

$$\frac{2x^2 + 3x + 4}{x}$$

Answer

Given,

$f(x) = \frac{2x^2 + 3x + 4}{x}$

$\Rightarrow f(x) = 2x + 3 + \frac{4}{x}$

$\Rightarrow f(x) = 2x + 3 + 4x^{-1}$

we need to find f′(x), so differentiating both sides with respect to x –

$\therefore \frac{d}{dx}\{f(x)\} = \frac{d}{dx}(2x + 3 + 4x^{-1})$

Using algebra of derivatives –

$\Rightarrow f'(x) = 2\frac{d}{dx}(x) + \frac{d}{dx}(3) + 4\frac{d}{dx}(x^{-1})$

Use the formula: $\frac{d}{dx}(x^n) = nx^{n-1}$ and $\frac{d}{dx}(\text{constant}) = 0$

$\therefore f'(x) = 2 + 0 + 4(-1)x^{-1-1}$

$\Rightarrow f'(x) = 2 - 4x^{-2}$

$\therefore f'(x) = 2 - 4x^{-2}$

10. Question

Differentiate the following with respect to x:

$$\frac{(x^3 + 1)(x - 2)}{x^2}$$

Answer

Given,

f(x) = $\frac{(x^3+1)(x-2)}{x^2}$

⇒ f(x) = $\frac{x^4-2x^3+x-2}{x^2}$

⇒ f(x) = $x^2 - 2x + x^{-1} - 2x^{-2}$

we need to find f'(x), so differentiating both sides with respect to x –

$\therefore \frac{d}{dx}\{f(x)\} = \frac{d}{dx}(x^2 - 2x + x^{-1} - 2x^{-2})$

Using algebra of derivatives –

⇒ f'(x) = $\frac{d}{dx}(x^2) - 2\frac{d}{dx}(x) + \frac{d}{dx}(x^{-1}) - 2\frac{d}{dx}(x^{-2})$

Use the formula: $\frac{d}{dx}(x^n) = nx^{n-1}$ and $\frac{d}{dx}(\text{constant}) = 0$

∴ f'(x) = $2x^{2-1} + 2x^{1-1} + (-1)x^{-1-1} - 2(-2)x^{-2-1}$

⇒ f'(x) = $2x + 2x^0 - 1x^{-2} + 4x^{-3}$

∴ f'(x) = $2x + 2 - x^{-2} + 4x^{-3}$

11. Question

Differentiate the following with respect to x:

$$\frac{a\cos x + b\sin x + c}{\sin x}$$

Answer

Given,

f(x) = $\frac{a\cos x + b\sin x + c}{\sin x}$

⇒ f(x) = $a\frac{\cos x}{\sin x} + b + \frac{c}{\sin x}$

⇒ f(x) = a cot x + b + c cosec x

we need to find f'(x), so differentiating both sides with respect to x –

$\therefore \frac{d}{dx}\{f(x)\} = \frac{d}{dx}$(a cot x + b + c cosec x)

Using algebra of derivatives –

⇒ f'(x) = $a\frac{d}{dx}(\cot x) + \frac{d}{dx}(b) + c\frac{d}{dx}(\operatorname{cosec} x)$

Use the formula: $\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$ & $\frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x \cot x$

∴ f'(x) = a(– cosec^2 x) + 0 + c(– cosec x cot x)

⇒ f'(x) = – a cosec^2 x – c cosec x cot x

∴ f'(x) = – a cosec^2 x – c cosec x cot x

12. Question

Differentiate the following with respect to x:

2 sec x + 3 cot x – 4 tan x

Answer

Given,

$f(x) = 2 \sec x + 3 \cot x - 4 \tan x$

we need to find f'(x), so differentiating both sides with respect to x –

$\therefore \frac{d}{dx}\{f(x)\} = \frac{d}{dx}(2 \sec x + 3 \cot x - 4 \tan x)$

Using algebra of derivatives –

$\Rightarrow f'(x) = 2\frac{d}{dx}(\sec x) + 3\frac{d}{dx}(\cot x) - 4\frac{d}{dx}(\tan x)$

Use the formula:

$\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x, \frac{d}{dx}(\sec x) = \sec x \tan x \;\&\; \frac{d}{dx}(\tan x) = \sec^2 x$

$\therefore f'(x) = 2(\sec x \tan x) + 3(-\operatorname{cosec}^2 x) - 4(\sec^2 x)$

$\Rightarrow f'(x) = 2\sec x \tan x - 3\operatorname{cosec}^2 x - 4\sec^2 x$

$\therefore f'(x) = 2\sec x \tan x - 3\operatorname{cosec}^2 x - 4\sec^2 x$

13. Question

Differentiate the following with respect to x:

$a_0 x^n + a_1 x^{n-1} + a_2 x^{n-2} + \ldots\ldots + a_{n-1} x + a_n$

Answer

Given,

$f(x) = a_0 x^n + a_1 x^{n-1} + a_2 x^{n-2} + \ldots\ldots + a_{n-1} x + a_n$

we need to find f'(x), so differentiating both sides with respect to x –

$\therefore \frac{d}{dx}\{f(x)\} = \frac{d}{dx}(a_0 x^n + a_1 x^{n-1} + a_2 x^{n-2} + \ldots\ldots + a_{n-1} x + a_n)$

Using algebra of derivatives –

$\Rightarrow f'(x) = \frac{d}{dx}(a_0 x^n) + a_1\frac{d}{dx}(x^{n-1}) + \ldots + a_{n-1}\frac{d}{dx}(x) + a_n\frac{d}{dx}(1)$

Use the formula: $\frac{d}{dx}(x^n) = nx^{n-1}$ and $\frac{d}{dx}(\text{constant}) = 0$

$\therefore f'(x) = a_0 n x^{n-1} + a_1 (n-1) x^{n-1-1} + a_2(n-2) x^{n-2-1} + \ldots\ldots + a_{n-1} + 0$

$\Rightarrow f'(x) = a_0 n x^{n-1} + a_1 (n-1) x^{n-2} + a_2(n-2) x^{n-3} + \ldots\ldots + a_{n-1}$

$\therefore f'(x) = a_0 n x^{n-1} + a_1 (n-1) x^{n-2} + a_2(n-2) x^{n-3} + \ldots\ldots + a_{n-1}$

14. Question

Differentiate the following with respect to x:

$$\frac{1}{\sin x} + 2^{x+3} + \frac{4}{\log_x 3}$$

Answer

Given,

$f(x) = \frac{1}{\sin x} + 2^{x+3} + \frac{4}{\log_x 3}$

using change of base formula for log, we can write –

$\log_x 3 = (\log_e 3)/(\log_e x)$

$\therefore f(x) = \operatorname{cosec} x + 2^3 2^x + 4 \frac{\log_e x}{\log_e 3}$

$\Rightarrow f(x) = \operatorname{cosec} x + 8.2^x + 4 \frac{\log_e x}{\log_e 3}$

we need to find f'(x), so differentiating both sides with respect to x –

$\therefore \frac{d}{dx}\{f(x)\} = \frac{d}{dx}(\operatorname{cosec} x + 8.2^x + 4 \frac{\log_e x}{\log_e 3})$

Using algebra of derivatives –

$\Rightarrow f'(x) = \frac{d}{dx}(\operatorname{cosec} x) + 8\frac{d}{dx}(2^x) + \frac{4}{\log_e 3}\frac{d}{dx}(\log_e x)$

Use the formula:

$\frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x \cot x, \frac{d}{dx}(a^x) = a^x \log_e a \ \& \frac{d}{dx}(\log_e x) = \frac{1}{x}$

$\therefore f'(x) = 2(-\operatorname{cosec} x \cot x) + 8(2^x \log_e 2) - 4/(\log_e 3)(1/x)$

$\Rightarrow f'(x) = -2\operatorname{cosec} x \cot x + 2^{x+3} \log_e 2 + \frac{4}{x \log_e 3}$

$\therefore f'(x) = -2\operatorname{cosec} x \cot x + 2^{x+3} \log_e 2 + \frac{4}{x \log_e 3}$

15. Question

Differentiate the following with respect to x:

$$\frac{(x+5)(2x^2-1)}{x}$$

Answer

Given,

$f(x) = \frac{(x+5)(2x^2-1)}{x}$

$\Rightarrow f(x) = \frac{2x^3 + 10x^2 - x - 5}{x}$

$\Rightarrow f(x) = 2x^2 + 10x - 1 - 5x^{-1}$

we need to find f'(x), so differentiating both sides with respect to x –

$\therefore \frac{d}{dx}\{f(x)\} = \frac{d}{dx}(2x^2 + 10x - 1 - 5x^{-1})$

Using algebra of derivatives –

$\Rightarrow f'(x) = 2\frac{d}{dx}(x^2) + 10\frac{d}{dx}(x) - \frac{d}{dx}(1) - 5\frac{d}{dx}(x^{-1})$

Use the formula: $\frac{d}{dx}(x^n) = nx^{n-1}$ and $\frac{d}{dx}(\text{constant}) = 0$

$\therefore f'(x) = 2(2x^{2-1}) + 10(1) - (-1)(0) - 5(-1)x^{-1-1}$

$\Rightarrow f'(x) = 4x + 10 + 0 + 5x^{-2}$

$\therefore f'(x) = 4x + 10 + 5x^{-2}$

16. Question

Differentiate the following with respect to x:

$$\log\left(\frac{1}{\sqrt{x}}\right) + 5x^a - 3a^x + \sqrt[3]{x^2} + 6\sqrt[4]{x^{-3}}$$

Answer

Given,

f(x) = $\log\left(\frac{1}{\sqrt{x}}\right) + 5x^a - 3a^x + \sqrt[3]{x^2} + 6\sqrt[4]{x^{-3}}$

⇒ f(x) = $\log\left(x^{-\frac{1}{2}}\right) + 5x^a - 3a^x + x^{\frac{2}{3}} + 6x^{-3/4}$

⇒ f(x) = – 0.5 log x + 5x^a – 3a^x + $x^{2/3}$ + 6$x^{-3/4}$

we need to find f'(x), so differentiating both sides with respect to x –

∴ $\frac{d}{dx}\{f(x)\} = \frac{d}{dx}\left(-0.5\log x + 5x^a - 3a^x + x^{\frac{2}{3}} + 6x^{-3/4}\right)$

Using algebra of derivatives –

⇒ f'(x) = $-\frac{1}{2}\frac{d}{dx}(\log x) + 5\frac{d}{dx}(x^a) - 3\frac{d}{dx}(a^x) + \frac{d}{dx}\left(x^{\frac{2}{3}}\right) + 6\frac{d}{dx}\left(x^{-\frac{3}{4}}\right)$

Use the formula:

$$\frac{d}{dx}(x^n) = nx^{n-1}, \frac{d}{dx}(\log_e x) = \frac{1}{x}, \frac{d}{dx}(a^x) = a^x \log_e a$$

∴ f'(x) = $-\frac{1}{2x} + 5ax^{a-1} - 3a^x\log_e a + \frac{2}{3}x^{\frac{2}{3}-1} + 6\left(-\frac{3}{4}\right)x^{-\frac{3}{4}-1}$

⇒ f'(x) = $-\frac{1}{2x} + 5ax^{a-1} - 3a^x\log_e a + \frac{2}{3}x^{-\frac{1}{3}} - \frac{9}{2}x^{-\frac{7}{4}}$

∴ f'(x) = $-\frac{1}{2x} + 5ax^{a-1} - 3a^x\log_e a + \frac{2}{3}x^{-\frac{1}{3}} - \frac{9}{2}x^{-\frac{7}{4}}$

17. Question

Differentiate the following with respect to x:

cos (x + a)

Answer

Given,

f(x) = cos (x + a)

Using cos (A + B) = cos A cos B – sin A sin B, we get –

∴ f(x) = cos x cos a – sin x sin a

we need to find f'(x), so differentiating both sides with respect to x –

∴ $\frac{d}{dx}\{f(x)\} = \frac{d}{dx}(\cos x \cos a - \sin x \sin a)$

Using algebra of derivatives –

⇒ f'(x) = $\frac{d}{dx}(\cos x \cos a) - \frac{d}{dx}(\sin x \sin a)$

As cos a and sin a are constants, so using algebra of derivatives we have –

⇒ f'(x) = $\cos a\frac{d}{dx}(\cos x) - \sin a\frac{d}{dx}(\sin x)$

Use the formula:

$\frac{d}{dx}(\cos x) = -\sin x$ & $\frac{d}{dx}\sin x = \cos x$

∴ f'(x) = – sin x cos a – sin a cos x

⇒ f'(x) = – (sin x cos a + sin a cos x)

Using sin (A + B) = sin A cos B + cos A sin B, we get –

∴ f'(x) = – sin (x + a)

18. Question

Differentiate the following with respect to x:

$$\frac{\cos(x-2)}{\sin x}$$

Answer

Given,

f(x) = $\frac{\cos(x-2)}{\sin x}$

Using cos (A + B) = cos A cos B – sin A sin B, we get –

∴ f(x) = $\frac{\cos x \cos 2 - \sin x \sin 2}{\sin x}$

⇒ f(x) = cos 2 cot x – sin 2

we need to find f'(x), so differentiating both sides with respect to x –

∴ $\frac{d}{dx}\{f(x)\} = \frac{d}{dx}(\cot x \cos 2 - \sin 2)$

Using algebra of derivatives –

⇒ f'(x) = $\frac{d}{dx}(\cot x \cos 2) - \frac{d}{dx}(\sin 2)$

As cos a and sin a are constants, so using algebra of derivatives we have –

⇒ f'(x) = $\cos 2 \frac{d}{dx}(\cot x) - \sin 2 \frac{d}{dx}(1)$

Use the formula:

$$\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$$

∴ f'(x) = – cosec^2 x cos 2 – sin 2 (0)

⇒ f'(x) = – cosec^2 x cos 2 – 0

∴ f'(x) = – cosec^2 x cos 2

19. Question

If $y = \left(\sin\frac{x}{2} + \cos\frac{x}{2}\right)^2$, find $\frac{dy}{dx}$ at $x = \frac{\pi}{6}$

Answer

Given,

y = $\left(\sin\frac{x}{2} + \cos\frac{x}{2}\right)^2$

Using (a + b)^2 = a^2 + 2ab + b^2

$y = \sin^2\frac{x}{2} + 2\sin\frac{x}{2}\cos\frac{x}{2} + \cos^2\frac{x}{2}$

$\Rightarrow y = 1 + 2\sin\frac{x}{2}\cos\frac{x}{2}$ {∵ $\sin^2 A + \cos^2 A = 1$ & 2sin A cos A = sin 2A}

⇒ y = 1 + sin x

Now, differentiating both sides w.r.t x –

$\therefore \frac{dy}{dx} = \frac{d}{dx}(1 + \sin x)$

Using algebra of derivatives –

$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}(1) + \frac{d}{dx}(\sin x)$

Use: $\frac{d}{dx}(\text{constant}) = 0$ & $\frac{d}{dx}(\sin x) = \cos x$

$\therefore \frac{dy}{dx} = 0 + \cos x = \cos x$

Hence, dy/dx at x = π/6 is

$\left(\frac{dy}{dx}\right)_{at\ x=\frac{\pi}{6}} = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$ ... ans

20. Question

If $y = \left(\frac{2 - 3\cos x}{\sin x}\right)$, find $\frac{dy}{dx}$ at $x = \frac{\pi}{4}$

Answer

Given,

$y = \frac{2 - 3\cos x}{\sin x}$

$y = \frac{2}{\sin x} - 3\frac{\cos x}{\sin x}$

⇒ y = 2 cosec x – 3 cot x

Now, differentiating both sides w.r.t x –

$\therefore \frac{dy}{dx} = \frac{d}{dx}(2\ \mathrm{cosec}\ x - 3\cot x)$

Using algebra of derivatives –

$\Rightarrow \frac{dy}{dx} = 2\frac{d}{dx}(\mathrm{cosec}\ x) - 3\frac{d}{dx}(\cot x)$

Use: $\frac{d}{dx}(\mathrm{cosec}\ x) = -\mathrm{cosec}\ x \cot x$ & $\frac{d}{dx}(\cot x) = -\mathrm{cosec}^2 x$

$\therefore \frac{dy}{dx} = -2\mathrm{cosec}\ x \cot x - 3(-\mathrm{cosec}^2 x)$

Hence, dy/dx at x = π/4 is

$\left(\frac{dy}{dx}\right)_{at\ x=\frac{\pi}{4}} = -2\mathrm{cosec}\left(\frac{\pi}{4}\right)\cot\left(\frac{\pi}{4}\right) + 3\ \mathrm{cosec}^2\left(\frac{\pi}{4}\right) = -2\sqrt{2} + 6$ ... ans

21. Question

Find the slope of the tangent to the curve $f(x) = 2x^6 + x^4 - 1$ at x = 1.

Answer

Given,

$y = 2x^6 + x^4 - 1$

We need to find slope of tangent of f(x) at x = 1.

Slope of the tangent is given by value of derivative at that point. So we need to find dy/dx first.

As, $y = 2x^6 + x^4 - 1$

Now, differentiating both sides w.r.t x –

$\therefore \frac{dy}{dx} = \frac{d}{dx}(2x^6 + x^4 - 1)$

Using algebra of derivatives –

$\Rightarrow \frac{dy}{dx} = 2\frac{d}{dx}(x^6) + \frac{d}{dx}(x^4) - \frac{d}{dx}(1)$

Use: $\frac{d}{dx}(x^n) = nx^{n-1}$ & $\frac{d}{dx}(\text{constant}) = 0$

$\therefore \frac{dy}{dx} = 2(6)x^{6-1} + 4x^{4-1} - 0$

$\Rightarrow \frac{dy}{dx} = 12x^5 + 4x^3 - 0$

As, slope of tangent at x = 1 will be given by the value of dy/dx at x = 1

$\therefore \left(\frac{dy}{dx}\right)_{\text{at } x=1} = 12(1^5) + 4(1^3) = 16$ ... ans

22. Question

If $\sqrt{\frac{x}{a}} + \sqrt{\frac{a}{x}}$, prove that: $2xy\frac{dy}{dx} = \left(\frac{x}{a} - \frac{a}{x}\right)$

Answer

Given,

$y = \sqrt{\frac{x}{a}} + \sqrt{\frac{a}{x}}$

We need to prove: $2x\frac{dy}{dx} = \left(\frac{x}{a} - \frac{a}{x}\right)$

As, $y = \sqrt{\frac{x}{a}} + \sqrt{\frac{a}{x}}$ ....equation 1

Now, differentiating both sides w.r.t x –

$\therefore \frac{dy}{dx} = \frac{d}{dx}(\sqrt{\frac{x}{a}} + \sqrt{\frac{a}{x}})$

Using algebra of derivatives –

$\Rightarrow \frac{dy}{dx} = \frac{1}{\sqrt{a}}\frac{d}{dx}\left(x^{\frac{1}{2}}\right) + \sqrt{a}\frac{d}{dx}\left(x^{-\frac{1}{2}}\right)$

Use: $\frac{d}{dx}(x^n) = nx^{n-1}$

$\therefore \frac{dy}{dx} = \frac{1}{\sqrt{a}}\left(\frac{1}{2}\right)x^{\frac{1}{2}-1} + \sqrt{a}\left(-\frac{1}{2}\right)x^{-\frac{1}{2}-1}$

$\Rightarrow \frac{dy}{dx} = \frac{1}{2\sqrt{a}}x^{-\frac{1}{2}} - \frac{\sqrt{a}}{2}x^{-\frac{3}{2}}$

$\Rightarrow 2\frac{dy}{dx} = \frac{1}{\sqrt{a}\sqrt{x}} - \frac{\sqrt{a}}{x\sqrt{x}}$

Multiplying x both sides –

$$\Rightarrow 2x\frac{dy}{dx} = \frac{x}{\sqrt{a}\sqrt{x}} - \frac{\sqrt{a}}{\sqrt{x}}$$

$$\Rightarrow 2x\frac{dy}{dx} = \frac{\sqrt{x}}{\sqrt{a}} - \frac{\sqrt{a}}{\sqrt{x}}$$

Now, multiplying y both sides –

$$\Rightarrow 2xy\frac{dy}{dx} = y\left(\frac{\sqrt{x}}{\sqrt{a}} - \frac{\sqrt{a}}{\sqrt{x}}\right)$$

$$\Rightarrow 2xy\frac{dy}{dx} = \left(\frac{\sqrt{x}}{\sqrt{a}} + \frac{\sqrt{a}}{\sqrt{x}}\right)\left(\frac{\sqrt{x}}{\sqrt{a}} - \frac{\sqrt{a}}{\sqrt{x}}\right)$$ {from equation 1}

Using $(a + b)(a - b) = a^2 - b^2$

$$\Rightarrow 2xy\frac{dy}{dx} = \left(\frac{\sqrt{x}}{\sqrt{a}}\right)^2 - \left(\frac{\sqrt{a}}{\sqrt{x}}\right)^2$$

$$\Rightarrow 2xy\frac{dy}{dx} = \left(\frac{x}{a} - \frac{a}{x}\right) \text{... ... proved}$$

23. Question

Find the rate at which the function $f(x) = x^4 - 2x^3 + 3x^2 + x + 5$ changes with respect to x.

Answer

Given,

$y = x^4 - 2x^3 + 3x^2 + x + 5$

We need to rate of change of f(x) w.r.t x.

Rate of change of a function w.r.t a given variable is obtained by differentiating the function w.r.t that variable only.

So in this case we will be finding dy/dx

As, $y = x^4 - 2x^3 + 3x^2 + x + 5$

Now, differentiating both sides w.r.t x –

$$\therefore \frac{dy}{dx} = \frac{d}{dx}(x^4 - 2x^3 + 3x^2 + x + 5)$$

Using algebra of derivatives –

$$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}(x^4) - 2\frac{d}{dx}(x^3) + 3\frac{d}{dx}(x^2) + \frac{d}{dx}(x) + \frac{d}{dx}(5)$$

Use: $\frac{d}{dx}(x^n) = nx^{n-1}$ & $\frac{d}{dx}(\text{constant}) = 0$

$$\therefore \frac{dy}{dx} = 4x^{4-1} - 2(3)x^{3-1} + 3(2)x^{2-1} + 1 + 0$$

$$\Rightarrow \frac{dy}{dx} = 4x^3 - 6x^2 + 6x + 1$$

∴ Rate of change of y w.r.t x is given by – $\frac{dy}{dx} = 4x^3 - 6x^2 + 6x + 1$

24. Question

If $y = \frac{2x^9}{3} - \frac{5}{7}x^7 + 6x^3 - x$, find $\frac{dy}{dx}$ at $x = 1$

Answer

Given,

$y = \frac{2x^9}{3} - \frac{5}{7}x^7 + 6x^3 - x$

We need to find dy/dx at x = 1

As, $y = \frac{2x^9}{3} - \frac{5}{7}x^7 + 6x^3 - x$

Now, differentiating both sides w.r.t x –

$\therefore \frac{dy}{dx} = \frac{d}{dx}(\frac{2x^9}{3} - \frac{5}{7}x^7 + 6x^3 - x)$

Using algebra of derivatives –

$\Rightarrow \frac{dy}{dx} = \frac{2}{3}\frac{d}{dx}(x^9) - \frac{5}{7}\frac{d}{dx}(x^7) + 6\frac{d}{dx}(x^3) - \frac{d}{dx}(x)$

Use: $\frac{d}{dx}(x^n) = nx^{n-1}$ & $\frac{d}{dx}(\text{constant}) = 0$

$\therefore \frac{dy}{dx} = \frac{2}{3}(9)x^{9-1} - \frac{5}{7}(7)x^{7-1} + 6(3)x^{3-1} - 1$

$\Rightarrow \frac{dy}{dx} = 6x^8 - 5x^6 + 18x^2 - 1$

$\therefore \left(\frac{dy}{dx}\right)_{\text{at } x=1} = 6(1^8) - 5(1^6) + 18(1^2) - 1 = 18$ ... ans

25. Question

If for f(x) = λ x^2 + μx + 12, f′ (4) = 15 and f′ (2) = 11, then find λ and μ.

Answer

Given,

y = λ x^2 + μx + 12

Now, differentiating both sides w.r.t x –

$\therefore \frac{dy}{dx} = \frac{d}{dx}(\lambda x^2 + \mu x + 12)$

Using algebra of derivatives –

$\Rightarrow \frac{dy}{dx} = \lambda\frac{d}{dx}(x^2) + \mu\frac{d}{dx}(x) + \frac{d}{dx}(12)$

Use: $\frac{d}{dx}(x^n) = nx^{n-1}$ & $\frac{d}{dx}(\text{constant}) = 0$

$\therefore \frac{dy}{dx} = \lambda(2)x^{2-1} + \mu = 2\lambda x + \mu$

Now, we have –

f′(x) = 2λx + μ

Given,

f′(4) = 15

⇒ 2λ (4) + μ = 15

⇒ 8λ + μ = 15 ......equation 1

Also f′(2) = 11

⇒ 2λ(2) + μ = 11

⇒ 4λ + μ = 11 .....equation 2

Subtracting equation 2 from equation 1, we have –

$4\lambda = 15 - 11 = 4$

$\therefore \lambda = 1$

Putting $\lambda = 1$ in equation 2

$4 + \mu = 11$

$\therefore \mu = 7$

Hence,

$\lambda = 1$ & $\mu = 7$

26. Question

For the function $f(x) = \frac{x^{100}}{100} + \frac{x^{99}}{99} + ... + \frac{x^2}{2} + x + 1$. Prove that f'(1) = 100 f' (0).

Answer

Given,

$$f(x) = \frac{x^{100}}{100} + \frac{x^{99}}{99} + ... + \frac{x^2}{2} + x + 1$$

Now, differentiating both sides w.r.t x –

$$\therefore \frac{d}{dx}\{f(x)\} = \frac{d}{dx}\left(\frac{x^{100}}{100} + \frac{x^{99}}{99} + ... + \frac{x^2}{2} + x + 1\right)$$

Using algebra of derivatives –

$$\Rightarrow f'(x) = \frac{1}{100}\frac{d}{dx}(x^{100}) + \frac{1}{99}\frac{d}{dx}(x^{99}) + ... + \frac{1}{2}\frac{d}{dx}(x^2) + \frac{d}{dx}(x) + \frac{d}{dx}(1)$$

Use: $\frac{d}{dx}(x^n) = nx^{n-1}$ & $\frac{d}{dx}(\text{constant}) = 0$

$$\therefore f'(x) = \frac{100}{100}x^{99} + \frac{99}{99}x^{98} + ... + \frac{2}{2}x + 1 + 0$$

$\Rightarrow f'(x) = x^{99} + x^{98} + ... + x + 1$

$\therefore f'(1) = 1^{99} + 1^{98} + ... + 1 + 1$ (sum of total 100 ones) = 100

$\therefore f'(1) = 100$

As, $f'(0) = 0 + 0 + ... + 0 + 1 = 1$

$\therefore$ we can write as

$f'(1) = 100\times1 = 100\times f'(0)$

Hence,

$f'(1) = 100 f'(0)$ ....proved

Exercise 30.4

1. Question

Differentiate the following functions with respect to x:

$x^3 \sin x$

Answer

Let, $y = x^3 \sin x$

We have to find dy/dx

As we can observe that y is a product of two functions say u and v where,

$u = x^3$ and $v = \sin x$

$\therefore y = uv$

As we know that to find the derivative of product of two function we apply product rule of differentiation.

By product rule, we have –

$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$ ...equation 1

As, $u = x^3$

$\therefore \frac{du}{dx} = 3x^{3-1} = 3x^2$ ...equation 2 $\{\because \frac{d}{dx}(x^n) = nx^{n-1}\}$

As, $v = \sin x$

$\therefore \frac{dv}{dx} = \frac{d}{dx}(\sin x) = \cos x$ ...equation 3 $\{\because \frac{d}{dx}(\sin x) = \cos x\}$

∴ from equation 1, we can find dy/dx

$\therefore \frac{dy}{dx} = x^3\frac{dv}{dx} + \sin x\,\frac{du}{dx}$

$\Rightarrow \frac{dy}{dx} = x^3\cos x + 3x^2\sin x$ {using equation 2 & 3}

Hence,

$$\frac{dy}{dx} = x^3\cos x + 3x^2\sin x \;\ldots.\text{ans}$$

2. Question

Differentiate the following functions with respect to x:

$x^3 e^x$

Answer

Let, $y = x^3 e^x$

We have to find dy/dx

As we can observe that y is a product of two functions say u and v where,

$u = x^3$ and $v = e^x$

$\therefore y = uv$

As we know that to find the derivative of product of two function we apply product rule of differentiation.

By product rule, we have –

$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$ ...equation 1

As, $u = x^3$

$\therefore \frac{du}{dx} = 3x^{3-1} = 3x^2$ ...equation 2 $\{\because \frac{d}{dx}(x^n) = nx^{n-1}\}$

As, $v = e^x$

$\therefore \frac{dv}{dx} = \frac{d}{dx}(e^x) = e^x$ ...equation 3 $\{\because \frac{d}{dx}(e^x) = e^x\}$

∴ from equation 1, we can find dy/dx

$\therefore \frac{dy}{dx} = x^3\frac{dv}{dx} + e^x\frac{du}{dx}$

$\Rightarrow \frac{dy}{dx} = x^3e^x + 3x^2e^x$ {using equation 2 & 3}

Hence,

$$\frac{dy}{dx} = x^2e^x(x + 3) \text{ ....ans}$$

3. Question

Differentiate the following functions with respect to x:

x^2 e^x log x

Answer

Let, y = x^2 e^x log x

We have to find dy/dx

As we can observe that y is a product of three functions say u, v & w where,

u = x^2

v = e^x

w = log x

∴ y = uvw

As we know that to find the derivative of product of three function we apply product rule of differentiation.

By product rule, we have –

$\frac{dy}{dx} = uw\frac{dv}{dx} + vw\frac{du}{dx} + uv\frac{dw}{dx}$ ...equation 1

As, u = x^2

$\therefore \frac{du}{dx} = 2x^{2-1} = 2x$ ...equation 2 $\{\because \frac{d}{dx}(x^n) = nx^{n-1}\}$

As, v = e^x

$\therefore \frac{dv}{dx} = \frac{d}{dx}(e^x) = e^x$ ...equation 3 $\{\because \frac{d}{dx}(e^x) = e^x\}$

As, w = log x

$\therefore \frac{dw}{dx} = \frac{d}{dx}(\log x) = \frac{1}{x}$ ...equation 4 $\{\because \frac{d}{dx}(\log_e x) = \frac{1}{x}\}$

∴ from equation 1, we can find dy/dx

$\therefore \frac{dy}{dx} = x^2\log x\frac{dv}{dx} + e^x\log x\frac{du}{dx} + x^2e^x\frac{dw}{dx}$

$\Rightarrow \frac{dy}{dx} = x^2e^x\log x + 2xe^x\log x + x^2e^x\frac{1}{x}$ {using equation 2, 3 & 4}

Hence,

$$\frac{dy}{dx} = xe^x(x\log x + 2\log x + 1) \text{ ....ans}$$

4. Question

Differentiate the following functions with respect to x:

x^n tan x

Answer

Let, $y = x^n \tan x$

We have to find dy/dx

As we can observe that y is a product of two functions say u and v where,

$u = x^n$ and $v = \tan x$

$\therefore y = uv$

As we know that to find the derivative of product of two function we apply product rule of differentiation.

By product rule, we have –

$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$ ...equation 1

As, $u = x^n$

$\therefore \frac{du}{dx} = nx^{n-1}$ ...equation 2 $\{\because \frac{d}{dx}(x^n) = nx^{n-1}\}$

As, $v = \tan x$

$\therefore \frac{dv}{dx} = \frac{d}{dx}(\tan x) = \sec^2 x$ ...equation 3 $\{\because \frac{d}{dx}(\tan x) = \sec^2 x\}$

$\therefore$ from equation 1, we can find dy/dx

$\therefore \frac{dy}{dx} = x^n\frac{dv}{dx} + \tan x\frac{du}{dx}$

$\Rightarrow \frac{dy}{dx} = x^n \sec^2 x + nx^{n-1}\tan x$ {using equation 2 & 3}

Hence,

$$\frac{dy}{dx} = x^n \sec^2 x + nx^{n-1}\tan x \text{ ....ans}$$

5. Question

Differentiate the following functions with respect to x:

$x^n \log_a x$

Answer

Let, $y = x^n \log_a x$

We have to find dy/dx

As we can observe that y is a product of two functions say u and v where,

$u = x^n$ and $v = \log_a x$

$\therefore y = uv$

As we know that to find the derivative of product of two function we apply product rule of differentiation.

By product rule, we have –

$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$ ...equation 1

As, $u = x^n$

$\therefore \frac{du}{dx} = nx^{n-1}$ ...equation 2 $\{\because \frac{d}{dx}(x^n) = nx^{n-1}\}$

As, $v = \log_a x$

$\therefore \frac{dv}{dx} = \frac{d}{dx}(\log_a x) = \frac{1}{x\log_e a}$ ...equation 3 $\{\because \frac{d}{dx}(\log_a x) = \frac{1}{x\log_e a}\}$

∴ from equation 1, we can find dy/dx

$\therefore \frac{dy}{dx} = x^n \frac{dv}{dx} + \log_a x \frac{du}{dx}$

$\Rightarrow \frac{dy}{dx} = x^n \frac{1}{x\log_e a} + nx^{n-1}\log_a x$ {using equation 2 & 3}

Hence,

$$\frac{dy}{dx} = x^{n-1}\left(\frac{1}{\log_e a} + n\log_a x\right) \text{....ans}$$

6. Question

Differentiate the following functions with respect to x:

$(x^3 + x^2 + 1) \sin x$

Answer

Let, $y = (x^3 + x^2 + 1) \sin x$

We have to find dy/dx

As we can observe that y is a product of two functions say u and v where,

$u = x^3 + x^2 + 1$ and $v = \sin x$

∴ y = uv

As we know that to find the derivative of product of two function we apply product rule of differentiation.

By product rule, we have –

$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$ ...equation 1

As, $u = x^3 + x^2 + 1$

$\therefore \frac{du}{dx} = \frac{d}{dx}(x^3 + x^2 + 1)$

$\Rightarrow \frac{du}{dx} = 3x^2 + 2x$ ...equation 2 $\{\because \frac{d}{dx}(x^n) = nx^{n-1}\}$

As, $v = \sin x$

$\therefore \frac{dv}{dx} = \frac{d}{dx}(\sin x) = \cos x$ ...equation 3 $\{\because \frac{d}{dx}(\sin x) = \cos x\}$

∴ from equation 1, we can find dy/dx

$\therefore \frac{dy}{dx} = (x^3 + x^2 + 1)\frac{dv}{dx} + \sin x \frac{du}{dx}$

$\Rightarrow \frac{dy}{dx} = (x^3 + x^2 + 1)\cos x + (3x^2 + 2x)\sin x$ {using equation 2 & 3}

Hence,

$$\frac{dy}{dx} = (x^3 + x^2 + 1)\cos x + (3x^2 + 2x)\sin x \text{ ....ans}$$

7. Question

Differentiate the following functions with respect to x:

cos x sin x

Answer

Let, y = cos x sin x

We have to find dy/dx

As we can observe that y is a product of two functions say u and v where,

u = cos x and v = sin x

∴ y = uv

As we know that to find the derivative of product of two function we apply product rule of differentiation.

By product rule, we have –

$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$ ...equation 1

As, u = cos x

∴ $\frac{du}{dx} = \frac{d}{dx}(\cos x) = -\sin x$ ...equation 2 {∵ $\frac{d}{dx}(\cos x) = -\sin x$ }

As, v = sin x

∴ $\frac{dv}{dx} = \frac{d}{dx}(\sin x) = \cos x$ ...equation 3 {∵ $\frac{d}{dx}(\sin x) = \cos x$}

∴ from equation 1, we can find dy/dx

∴ $\frac{dy}{dx} = \cos x\frac{dv}{dx} + \sin x\frac{du}{dx}$

⇒ $\frac{dy}{dx} = \cos x(\cos x) + \sin x(-\sin x)$ {using equation 2 & 3}

⇒ $\frac{dy}{dx} = \cos^2 x - \sin^2 x$

Hence,

$\frac{dy}{dx} = \cos^2 x - \sin^2 x$ ....ans

8. Question

Differentiate the following functions with respect to x:

$$\frac{2^x \cot x}{\sqrt{x}}$$

Answer

Let, y = $\frac{2^x \cot x}{\sqrt{x}} = 2^x \cot x\, x^{-\frac{1}{2}}$

We have to find dy/dx

As we can observe that y is a product of three functions say u, v & w where,

u = $x^{-1/2}$

v = 2^x

w = cot x

∴ y = uvw

As we know that to find the derivative of product of three function we apply product rule of differentiation.

By product rule, we have –

$\frac{dy}{dx} = uw\frac{dv}{dx} + vw\frac{du}{dx} + uv\frac{dw}{dx}$ ...equation 1

As, u = $x^{-1/2}$

$\therefore \frac{du}{dx} = -\frac{1}{2}x^{-\frac{1}{2}-1} = -\frac{1}{2}x^{-\frac{3}{2}}$ ...equation 2 $\{\because \frac{d}{dx}(x^n) = nx^{n-1}\}$

As, v = 2^x

$\therefore \frac{dv}{dx} = \frac{d}{dx}(2^x) = 2^x \log_e 2$ ...equation 3 $\{\because \frac{d}{dx}(a^x) = a^x \log a\}$

As, w = cot x

$\therefore \frac{dw}{dx} = \frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$ ...equation 4 $\{\because \frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x\}$

∴ from equation 1, we can find dy/dx

$\therefore \frac{dy}{dx} = x^{-\frac{1}{2}} \cot x \frac{dv}{dx} + 2^x \cot x \frac{du}{dx} + x^{-\frac{1}{2}} 2^x \frac{dw}{dx}$

using equation 2, 3 & 4,we have –

$\Rightarrow \frac{dy}{dx} = x^{-\frac{1}{2}} \cot x\, 2^x \log 2 + 2^x \cot x \left(-\frac{1}{2}x^{-\frac{3}{2}}\right) + x^{-\frac{1}{2}} 2^x(-\operatorname{cosec}^2 x)$

Hence,

$$\frac{dy}{dx} = 2^x x^{-\frac{1}{2}}\left(\log 2 \cot x - \frac{\cot x}{2x} - \operatorname{cosec}^2 x\right) \text{ .... ans}$$

9. Question

Differentiate the following functions with respect to x:

$x^2 \sin x \log x$

Answer

Let, y = $x^2 \sin x \log x$

We have to find dy/dx

As we can observe that y is a product of three functions say u, v & w where,

u = x^2

v = sin x

w = log x

∴ y = uvw

As we know that to find the derivative of product of three function we apply product rule of differentiation.

By product rule, we have –

$\frac{dy}{dx} = uw\frac{dv}{dx} + vw\frac{du}{dx} + uv\frac{dw}{dx}$ ...equation 1

As, u = x^2

$\therefore \frac{du}{dx} = 2x^{2-1} = 2x$ ...equation 2 $\{\because \frac{d}{dx}(x^n) = nx^{n-1}\}$

As, v = sin x

$\therefore \frac{dv}{dx} = \frac{d}{dx}(\sin x) = \cos x$ ...equation 3 $\{\because \frac{d}{dx}(\sin x) = \cos x\}$

As, w = log x

$\therefore \frac{dw}{dx} = \frac{d}{dx}(\log x) = \frac{1}{x}$ ...equation 4 $\{\because \frac{d}{dx}(\log_e x) = \frac{1}{x}\}$

∴ from equation 1, we can find dy/dx

$\therefore \frac{dy}{dx} = x^2 \log x \frac{dv}{dx} + \sin x \log x \frac{du}{dx} + x^2 \sin x \frac{dw}{dx}$

using equation 2, 3 & 4, we have –

$\Rightarrow \frac{dy}{dx} = x^2 \cos x \log x + 2x \sin x \log x + x^2 \sin x \frac{1}{x}$

Hence,

$$\frac{dy}{dx} = x^2 \cos x \log x + 2x \sin x \log x + x \sin x \text{ .... ans}$$

10. Question

Differentiate the following functions with respect to x:

$x^5 e^x + x^6 \log x$

Answer

Let, $y = x^5 e^x + x^6 \log x$

Let, $A = x^5 e^x$ and $B = x^6 \log x$

$\therefore y = A + B$

$\Rightarrow \frac{dy}{dx} = \frac{dA}{dx} + \frac{dB}{dx}$

We have to find dA/dx first

As we can observe that A is a product of two functions say u and v where,

$u = x^5$ and $v = e^x$

$\therefore A = uv$

As we know that to find the derivative of product of two function we apply product rule of differentiation.

By product rule, we have –

$\frac{dA}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$ ...equation 1

As, $u = x^5$

$\therefore \frac{du}{dx} = 5x^{5-1} = 5x^4$ ...equation 2 $\{\because \frac{d}{dx}(x^n) = nx^{n-1}\}$

As, $v = e^x$

$\therefore \frac{dv}{dx} = \frac{d}{dx}(e^x) = e^x$ ...equation 3 $\{\because \frac{d}{dx}(e^x) = e^x\}$

∴ from equation 1, we can find dy/dx

$\therefore \frac{dA}{dx} = x^5 \frac{dv}{dx} + e^x \frac{du}{dx}$

$\Rightarrow \frac{dA}{dx} = x^5 e^x + 5x^4 e^x$ {using equation 2 & 3}

Hence,

$\frac{dA}{dx} = x^4 e^x(x + 5)$ .....equation 4

Now, we will find dB/dx first

As we can observe that A is a product of two functions say m and n where,

$m = x^6$ and $n = \log x$

$\therefore B = mn$

As we know that to find the derivative of product of two function we apply product rule of differentiation.

By product rule, we have –

$\frac{dB}{dx} = m\frac{dn}{dx} + m\frac{dm}{dx}$...equation 5

As, $m = x^6$

$\therefore \frac{dm}{dx} = 6x^{6-1} = 6x^5$ ...equation 6 $\{\because \frac{d}{dx}(x^n) = nx^{n-1}\}$

As, $v = \log x$

$\therefore \frac{dv}{dx} = \frac{d}{dx}(\log x) = \frac{1}{x}$...equation 7 $\{\because \frac{d}{dx}(\log x) = \frac{1}{x}\}$

$\therefore$ from equation 5, we can find dy/dx

$\therefore \frac{dB}{dx} = x^6\frac{dn}{dx} + \log x\frac{dm}{dx}$

$\Rightarrow \frac{dB}{dx} = x^6\frac{1}{x} + 6x^5\log x$ {using equation 6 & 7}

Hence,

$\frac{dB}{dx} = x^5(1 + 6\log x)$ .....equation 8

As, $\frac{dy}{dx} = \frac{dA}{dx} + \frac{dB}{dx}$

$\therefore$ from equation 4 and 8, we have –

$$\frac{dy}{dx} = x^4e^x(x + 5) + x^5(1 + 6\log x) \text{ ....ans}$$

11. Question

Differentiate the following functions with respect to x:

(x sin x + cos x)(x cos x – sin x)

Answer

Let, y = (x sin x + cos x)(x cos x – sin x)

We have to find dy/dx

As we can observe that y is a product of two functions say u and v where,

u = x sin x + cos x and v = x cos x – sin x

$\therefore$ y = uv

As we know that to find the derivative of product of two function we apply product rule of differentiation.

By product rule, we have –

$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$...equation 1

As, u = x sin x + cos x

$\therefore \frac{du}{dx} = \frac{d}{dx}(x\sin x + \cos x)$

Using algebra of derivatives –

$\Rightarrow \frac{du}{dx} = \frac{d}{dx}(x\sin x) + \frac{d}{dx}(\cos x)$

$\because \frac{d}{dx}(\sin x) = \cos x \;\&\; \frac{d}{dx}(\cos x) = -\sin x$

$\therefore \frac{du}{dx} = x\frac{d}{dx}(\sin x) + \sin x\frac{d}{dx}(x) + \frac{d}{dx}(\cos x)$ {using product rule}

$\Rightarrow \frac{du}{dx} = x\cos x + \sin x - \sin x = x\cos x$ ...equation 2

As, v = x cos x – sin x

$$\therefore \frac{dv}{dx} = \frac{d}{dx}(x\cos x - \sin x)$$

Using algebra of derivatives –

$\Rightarrow \frac{dv}{dx} = \frac{d}{dx}(x\cos x) - \frac{d}{dx}(\sin x)$

$\because \frac{d}{dx}(\sin x) = \cos x \;\&\; \frac{d}{dx}(\cos x) = -\sin x$

$\therefore \frac{dv}{dx} = x\frac{d}{dx}(\cos x) + \cos x\frac{d}{dx}(x) - \frac{d}{dx}(\sin x)$ {using product rule}

$\Rightarrow \frac{dv}{dx} = -x\sin x + \cos x - \cos x = -x\sin x$ ...equation 3

$\therefore$ from equation 1, we can find dy/dx

$\therefore \frac{dy}{dx} = (x\sin x + \cos x)\frac{dv}{dx} + (x\cos x - \sin x)\frac{du}{dx}$

using equation 2 & 3, we get –

$\Rightarrow \frac{dy}{dx} = (x\sin x + \cos x)(-x\sin x) + (x\cos x - \sin x)(x\cos x)$

$\Rightarrow \frac{dy}{dx} = x^2(\cos^2 x - \sin^2 x) + x(\sin x\cos x + \cos x\sin x)$

As, we know that: $\cos^2 x - \sin^2 x = \cos 2x$ & $2\sin x\cos x = \sin 2x$

Hence,

$$\frac{dy}{dx} = x^2\cos 2x - x\sin 2x$$

12. Question

Differentiate the following functions with respect to x:

$(x\sin x + \cos x)(e^x + x^2\log x)$

Answer

Let, $y = (x\sin x + \cos x)(e^x + x^2\log x)$

We have to find dy/dx

As we can observe that y is a product of two functions say u and v where,

$u = x\sin x + \cos x$ and $v = (e^x + x^2\log x)$

$\therefore y = uv$

As we know that to find the derivative of product of two function we apply product rule of differentiation.

By product rule, we have –

$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$ ...equation 1

As, u = x sin x + cos x

$\therefore \frac{du}{dx} = \frac{d}{dx}(x \sin x + \cos x)$

Using algebra of derivatives –

$\Rightarrow \frac{du}{dx} = \frac{d}{dx}(x \sin x) + \frac{d}{dx}(\cos x)$

$\because \frac{d}{dx}(\sin x) = \cos x \;\&\; \frac{d}{dx}(\cos x) = -\sin x$

$\therefore \frac{du}{dx} = x\frac{d}{dx}(\sin x) + \sin x \frac{d}{dx}(x) + \frac{d}{dx}(\cos x)$ {using product rule}

$\Rightarrow \frac{du}{dx} = x\cos x + \sin x - \sin x = x \cos x$ ...equation 2

As, v = e^x + x^2 log x

$\therefore \frac{dv}{dx} = \frac{d}{dx}(e^x + x^2 \log x)$

Using algebra of derivatives –

$\Rightarrow \frac{dv}{dx} = \frac{d}{dx}(e^x) + \frac{d}{dx}(x^2 \log x)$

$\because \frac{d}{dx}(e^x) = e^x \;\&\; \frac{d}{dx}(\log x) = \frac{1}{x}$

$\therefore \frac{dv}{dx} = x^2\frac{d}{dx}(\log x) + \log x \frac{d}{dx}(x^2) + \frac{d}{dx}(e^x)$ {using product rule}

$\Rightarrow \frac{dv}{dx} = \frac{x^2}{x} + 2x \log x + e^x = x + 2x \log x + e^x$ ...equation 3

∴ from equation 1, we can find dy/dx

$\therefore \frac{dy}{dx} = (x \sin x + \cos x)\frac{dv}{dx} + (x^2 \log x + e^x)\frac{du}{dx}$

using equation 2 & 3, we get –

$\Rightarrow \frac{dy}{dx} = (x \sin x + \cos x)(x + 2x \log x + e^x) + (e^x + x^2 \log x)(x \cos x)$

$\Rightarrow \frac{dy}{dx} = (x \sin x + \cos x)(x + 2x \log x + e^x) + (e^x + x^2 \log x)(x \cos x)$

Hence,

$$\frac{dy}{dx} = (x \sin x + \cos x)(x + 2x \log x + e^x) + (e^x + x^2 \log x)(x \cos x)$$

13. Question

Differentiate the following functions with respect to x:

(1 – 2 tan x)(5 + 4 sin x)

Answer

Let, y = (1 – 2 tan x)(5 + 4 sin x)

We have to find dy/dx

As we can observe that y is a product of two functions say u and v where,

u = (1 – 2tan x) and v = (5 + 4sin x)

∴ y = uv

As we know that to find the derivative of product of two function we apply product rule of differentiation.

By product rule, we have –

$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$ ...equation 1

As, u = (1 – 2tan x)

$\therefore \frac{du}{dx} = \frac{d}{dx}(1 - 2\tan x)$

$\Rightarrow \frac{du}{dx} = \frac{d}{dx}(1) - 2\frac{d}{dx}(\tan x) = 0 - 2\sec^2 x$

$\Rightarrow \frac{du}{dx} = -2\sec^2 x$ .....equation 2 $\{\because \frac{d}{dx}(\tan x) = \sec^2 x\}$

As, v = 5 + 4sin x

$\therefore \frac{dv}{dx} = \frac{d}{dx}(5 + 4\sin x)$

$\Rightarrow \frac{dv}{dx} = \frac{d}{dx}(5) + 4\frac{d}{dx}(\sin x) = 0 + 4\cos x$

$\Rightarrow \frac{dv}{dx} = 4\cos x$ ...equation 3 $\{\because \frac{d}{dx}(\sin x) = \cos x\}$

∴ from equation 1, we can find dy/dx

$\therefore \frac{dy}{dx} = (1 - 2\tan x)\frac{dv}{dx} + (5 + 4\sin x)\frac{du}{dx}$

using equation 2 & 3, we get –

$\Rightarrow \frac{dy}{dx} = (1 - 2\tan x)(4\cos x) + (5 + 4\sin x)(-2\sec^2 x)$

$\Rightarrow \frac{dy}{dx} = 4\cos x - 8\tan x \times \cos x - 10\sec^2 x - 8\sin x \times \sec^2 x$

∵ sin x = tan x cos x , so we get –

$\Rightarrow \frac{dy}{dx} = 4\cos x - 8\sin x - 10\sec^2 x - 8\sec x\tan x$

Hence,

$$\frac{dy}{dx} = 4\cos x - 8\sin x - 10\sec^2 x - 8\sec x\tan x \text{ .... ans}$$

14. Question

Differentiate the following functions with respect to x:

$(x^2 + 1)\cos x$

Answer

Let, y = $(x^2 + 1)\cos x$

We have to find dy/dx

As we can observe that y is a product of two functions say u and v where,

u = $x^2 + 1$ and v = cos x

∴ y = uv

As we know that to find the derivative of product of two function we apply product rule of differentiation.

By product rule, we have –

$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$ ...equation 1

As, u = $x^2 + 1$

$\therefore \frac{du}{dx} = \frac{d}{dx}(x^2 + 1) = \frac{d}{dx}(x^2) + \frac{d}{dx}(1)$

$\Rightarrow \frac{du}{dx} = 2x$ ...equation 2 $\{\because \frac{d}{dx}(x^n) = nx^{n-1}\}$

As, v = cos x

$\therefore \frac{dv}{dx} = \frac{d}{dx}(\cos x) = -\sin x$ ...equation 3 $\{\because \frac{d}{dx}(\cos x) = -\sin x\}$

∴ from equation 1, we can find dy/dx

$\therefore \frac{dy}{dx} = (x^2 + 1)\frac{dv}{dx} + \cos x \frac{du}{dx}$

$\Rightarrow \frac{dy}{dx} = (x^2 + 1)(-\sin x) + 2x\cos x$ {using equation 2 & 3}

Hence,

$$\frac{dy}{dx} = (x^2 + 1)(-\sin x) + 2x\cos x \; \text{....ans}$$

15. Question

Differentiate the following functions with respect to x:

$\sin^2 x$

Answer

Let, y = $\sin^2 x$ = sin x sin x

We have to find dy/dx

As we can observe that y is a product of two functions say u and v where,

u = sin x and v = sin x

∴ y = uv

As we know that to find the derivative of product of two function we apply product rule of differentiation.

By product rule, we have –

$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$ ...equation 1

As, u = sin x

$\therefore \frac{du}{dx} = \frac{d}{dx}(\sin x) = \cos x$ ...equation 2 $\{\because \frac{d}{dx}(\sin x) = \cos x\}$

As, v = sin x

$\therefore \frac{dv}{dx} = \frac{d}{dx}(\sin x) = \cos x$ ...equation 3 $\{\because \frac{d}{dx}(\sin x) = \cos x\}$

∴ from equation 1, we can find dy/dx

$\therefore \frac{dy}{dx} = \sin x\frac{dv}{dx} + \sin x \frac{du}{dx}$

$\Rightarrow \frac{dy}{dx} = \sin x(\cos x) + \sin x(\cos x)$ {using equation 2 & 3}

$\Rightarrow \frac{dy}{dx} = 2\sin x \cos x$

Hence,

$$\frac{dy}{dx} = 2 \sin x \cos x = \sin 2x \ \ldots . \text{ans}$$

16. Question

Differentiate the following functions with respect to x:

$$\log_{x^2} x$$

Answer

Let, y = $\log_{x^2} x$

Using change of base formula for logarithm, we can write y as –

$$y = \frac{\log_e x}{\log_e x^2} = \frac{\log_e x}{2 \log_e x} = \frac{1}{2} = \text{constant}$$

As, y = 1/2

We know that $\frac{d}{dx}(\text{constant}) = 0$

$\therefore \frac{dy}{dx} = 0$ ...ans

17. Question

Differentiate the following functions with respect to x:

$$e^x \log \sqrt{x} \tan x$$

Answer

Let, y = e^x log √x tan x = e^x log $x^{1/2}$ tan x = 1/2 e^x log x tan x

We have to find dy/dx

As we can observe that y is a product of three functions say u, v & w where,

u = log x

v = e^x

w = tan x

∴ y = 1/2 uvw

As we know that to find the derivative of product of three function we apply product rule of differentiation.

By product rule, we have –

$\frac{dy}{dx} = \frac{1}{2}\left(uw\frac{dv}{dx} + vw\frac{du}{dx} + uv\frac{dw}{dx}\right)$ ...equation 1

As, u = log x

$\therefore \frac{du}{dx} = \frac{d}{dx}(\log x) = \frac{1}{x}$ ...equation 2 $\{\because \frac{d}{dx}(\log x) = \frac{1}{x}\}$

As, v = e^x

$\therefore \frac{dv}{dx} = \frac{d}{dx}(e^x) = e^x \log_e e = e^x$ ...equation 3 $\{\because \frac{d}{dx}(e^x) = e^x\}$

As, w = tan x

$\therefore \frac{dw}{dx} = \frac{d}{dx}(\tan x) = \sec^2 x$ ...equation 4 $\{\because \frac{d}{dx}(\tan x) = \sec^2 x\}$

∴ from equation 1, we can find dy/dx

$\therefore \frac{dy}{dx} = \frac{1}{2}(\log x \tan x \frac{dv}{dx} + e^x \tan x \frac{du}{dx} + \log x\, e^x \frac{dw}{dx})$

using equation 2, 3 & 4,we have –

$\Rightarrow \frac{dy}{dx} = \frac{1}{2}(\log x \tan x\, e^x + e^x \tan x \frac{1}{x} + e^x \log x \sec^2 x)$

Hence,

$$\frac{dy}{dx} = \frac{1}{2}e^x(\log x \tan x + \frac{\tan x}{x} + \log x \sec^2 x) \text{ ....ans}$$

18. Question

Differentiate the following functions with respect to x:

x^3 e^x cos x

Answer

Let, y = x^3 e^x cos x

We have to find dy/dx

As we can observe that y is a product of three functions say u, v & w where,

u = x^3

v = cos x

w = e^x

∴ y = uvw

As we know that to find the derivative of product of three function we apply product rule of differentiation.

By product rule, we have –

$\frac{dy}{dx} = uw\frac{dv}{dx} + vw\frac{du}{dx} + uv\frac{dw}{dx}$ ...equation 1

As, u = x^3

$\therefore \frac{du}{dx} = 3x^{3-1} = 3x^2$ ...equation 2 $\{\because \frac{d}{dx}(x^n) = nx^{n-1}\}$

As, v = cos x

$\therefore \frac{dv}{dx} = \frac{d}{dx}(\cos x) = -\sin x$ ...equation 3 $\{\because \frac{d}{dx}(\cos x) = -\sin x\}$

As, w = e^x

$\therefore \frac{dw}{dx} = \frac{d}{dx}(e^x) = e^x$ ...equation 4 $\{\because \frac{d}{dx}(e^x) = e^x\}$

∴ from equation 1, we can find dy/dx

$\therefore \frac{dy}{dx} = x^3 e^x \frac{dv}{dx} + \cos x\, e^x \frac{du}{dx} + x^3 \cos x \frac{dw}{dx}$

using equation 2, 3 & 4, we have –

$\Rightarrow \frac{dy}{dx} = x^3 e^x (-\sin x) + e^x \cos x\, (3x^2) + x^3 \cos x\, e^x$

Hence,

$$\frac{dy}{dx} = x^2 e^x(-x \sin x + 3 \cos x + x \cos x) \text{ ....ans}$$

19. Question

Differentiate the following functions with respect to x:

$$\frac{x^2 \cos\frac{\pi}{4}}{\sin x}$$

Answer

Let, $y = \frac{x^2 \cos\frac{\pi}{4}}{\sin x} = \frac{x^2}{\sqrt{2}\sin x} = \frac{1}{\sqrt{2}}x^2 \operatorname{cosec} x$ {∵ cos π/4 = 1/√2 }

We have to find dy/dx

As we can observe that y is a product of two functions say u and v where,

u = x^2 and v = cosec x

∴ y = (1/√2) uv

As we know that to find the derivative of product of two function we apply product rule of differentiation.

By product rule, we have –

$\frac{dy}{dx} = \frac{1}{\sqrt{2}}\left(u\frac{dv}{dx} + v\frac{du}{dx}\right)$ ...equation 1

As, u = x^2

∴ $\frac{du}{dx} = 2x^{2-1} = 2x$ ...equation 2 {∵ $\frac{d}{dx}(x^n) = nx^{n-1}$ }

As, v = cosec x

$$\therefore \frac{dv}{dx} = \frac{d}{dx}(\operatorname{cosec} x)$$

⇒ $\frac{dv}{dx} = = -\operatorname{cosec} x \cot x$ ...equation 3 {∵ $\frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x \cot x$}

∴ from equation 1, we can find dy/dx

∴ $\frac{dy}{dx} = \frac{1}{\sqrt{2}}\left(x^2\frac{dv}{dx} + \operatorname{cosec} x\frac{du}{dx}\right)$

⇒ $\frac{dy}{dx} = \frac{1}{\sqrt{2}}(-x^2 \operatorname{cosec} x \cot x + 2x \operatorname{cosec} x)$ {using equation 2 & 3}

Hence,

$$\frac{dy}{dx} = \frac{1}{\sqrt{2}}(-x^2 \operatorname{cosec} x \cot x + 2x \operatorname{cosec} x) \text{ .... ans}$$

20. Question

Differentiate the following functions with respect to x:

x^4 (5 sin x – 3 cos x)

Answer

Let, y = x^4 (5sin x – 3cos x)

We have to find dy/dx

As we can observe that y is a product of two functions say u and v where,

u = x^4 and v = 5sin x – 3cos x

∴ y = uv

As we know that to find the derivative of product of two function we apply product rule of differentiation.

By product rule, we have –

$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$ ...equation 1

As, u = x^4

$\therefore \frac{du}{dx} = 4x^{4-1} = 4x^3$ ...equation 2 $\{\because \frac{d}{dx}(x^n) = nx^{n-1}\}$

As, v = 5sin x – 3cos x

$$\therefore \frac{dv}{dx} = \frac{d}{dx}(5\sin x - 3\cos x) = 5\frac{d}{dx}(\sin x) - 3\frac{d}{dx}(\cos x)$$

Using: $\frac{d}{dx}(\sin x) = \cos x$ & $\frac{d}{dx}(\cos x) = -\sin x$

$\Rightarrow \frac{dv}{dx} = 5\cos x + 3\sin x$ ...equation 3

∴ from equation 1, we can find dy/dx

$\therefore \frac{dy}{dx} = x^4\frac{dv}{dx} + (5\sin x - 3\cos x)\frac{du}{dx}$

$\Rightarrow \frac{dy}{dx} = x^4(5\cos x + 3\sin x) + 4x^3(5\sin x - 3\cos x)$ {using equation 2 & 3}

Hence,

$$\frac{dy}{dx} = x^3(5x\cos x + 3x\sin x + 20\sin x - 12\cos x) \text{ .... ans}$$

21. Question

Differentiate the following functions with respect to x:

$(2x^2 – 3)$sin x

Answer

Let, y = $(2x^2 – 3)$ sin x

We have to find dy/dx

As we can observe that y is a product of two functions say u and v where,

u = $2x^2$ – 3 and v = sin x

∴ y = uv

As we know that to find the derivative of product of two function we apply product rule of differentiation.

By product rule, we have –

$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$ ...equation 1

As, u = $2x^2$ – 3

$\therefore \frac{du}{dx} = 4x$ ...equation 2 $\{\because \frac{d}{dx}(x^n) = nx^{n-1}\}$

As, v = sin x

$\therefore \frac{dv}{dx} = \frac{d}{dx}(\sin x) = \cos x$ ...equation 3 $\{\because \frac{d}{dx}(\sin x) = \cos x\}$

∴ from equation 1, we can find dy/dx

$\therefore \frac{dy}{dx} = (2x^2 - 3)\frac{dv}{dx} + \sin x\frac{du}{dx}$

$\Rightarrow \frac{dy}{dx} = (2x^2 - 3)\cos x + 4x\sin x$ {using equation 2 & 3}

Hence,

$$\frac{dy}{dx} = (2x^2 - 3)\cos x + 4x\sin x \;....\text{ans}$$

22. Question

Differentiate the following functions with respect to x:

$x^5 (3 - 6x^{-9})$

Answer

Let, $y = x^5 (3 - 6x^{-9})$

We have to find dy/dx

As we can observe that y is a product of two functions say u and v where,

$u = x^5$ and $v = 3 - 6x^{-9}$

$\therefore y = uv$

As we know that to find the derivative of product of two function we apply product rule of differentiation.

By product rule, we have –

$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$ ...equation 1

As, $u = x^5$

$\therefore \frac{du}{dx} = 5x^{5-1} = 5x^4$ ...equation 2 $\{\because \frac{d}{dx}(x^n) = nx^{n-1}\}$

As, $v = 3 - 6x^{-9}$

$\therefore \frac{dv}{dx} = \frac{d}{dx}(3 - 6x^{-9}) = 54\,x^{-10}$ ...equation 3 $\{\because \frac{d}{dx}(x^n) = nx^{n-1}\}$

$\therefore$ from equation 1, we can find dy/dx

$\therefore \frac{dy}{dx} = x^5\frac{dv}{dx} + (3 - 6x^{-9})\frac{du}{dx}$

$\Rightarrow \frac{dy}{dx} = x^5(54x^{-10}) + (3 - 6x^{-9})(5x^4)$ {using equation 2 & 3}

$\Rightarrow \frac{dy}{dx} = 54\,x^{-5} + 15\,x^4 - 30x^{-5} = 24x^{-5} + 15x^4$

Hence,

$$\frac{dy}{dx} = 24x^{-5} + 15x^4 \;....\text{ans}$$

23. Question

Differentiate the following functions with respect to x:

$x^{-4} (3 - 4x^{-5})$

Answer

Let, $y = x^{-4} (3 - 4x^{-5})$

We have to find dy/dx

As we can observe that y is a product of two functions say u and v where,

$u = x^{-4}$ and $v = 3 - 4x^{-5}$

$\therefore y = uv$

As we know that to find the derivative of product of two function we apply product rule of differentiation.

By product rule, we have –

$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$ ...equation 1

As, $u = x^{-4}$

$\therefore \frac{du}{dx} = (-4)x^{-4-1} = -4x^{-5}$ ...equation 2 $\{\because \frac{d}{dx}(x^n) = nx^{n-1}\}$

As, $v = 3 - 4x^{-5}$

$\therefore \frac{dv}{dx} = \frac{d}{dx}(3 - 4x^{-5}) = 20\,x^{-6}$ ...equation 3 $\{\because \frac{d}{dx}(x^n) = nx^{n-1}\}$

$\therefore$ from equation 1, we can find dy/dx

$\therefore \frac{dy}{dx} = x^{-4}\frac{dv}{dx} + (3 - 4x^{-5})\frac{du}{dx}$

$\Rightarrow \frac{dy}{dx} = x^{-4}(20x^{-6}) + (3 - 4x^{-5})(-4x^{-5})$ {using equation 2 & 3}

$\Rightarrow \frac{dy}{dx} = 20\,x^{-10} + 16\,x^{-10} - 12x^{-5} = 36x^{-10} - 12x^{-5}$

Hence,

$$\frac{dy}{dx} = 36x^{-10} - 12x^{-5} \;\ldots.\text{ans}$$

24. Question

Differentiate the following functions with respect to x:

$x^{-3}(5 + 3x)$

Answer

Let, $y = x^{-3}(5 + 3x)$

We have to find dy/dx

As we can observe that y is a product of two functions say u and v where,

$u = x^{-3}$ and $v = (5 + 3x)$

$\therefore y = uv$

As we know that to find the derivative of product of two function we apply product rule of differentiation.

By product rule, we have –

$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$ ...equation 1

As, $u = x^{-3}$

$\therefore \frac{du}{dx} = (-3)x^{-3-1} = -3x^{-4}$ ...equation 2 $\{\because \frac{d}{dx}(x^n) = nx^{n-1}\}$

As, $v = 5 + 3x$

$\therefore \frac{dv}{dx} = \frac{d}{dx}(5 + 3x) = 3$ ...equation 3 $\{\because \frac{d}{dx}(x^n) = nx^{n-1}\}$

$\therefore$ from equation 1, we can find dy/dx

$\therefore \frac{dy}{dx} = x^{-3}\frac{dv}{dx} + (5 + 3x)\frac{du}{dx}$

$\Rightarrow \frac{dy}{dx} = x^{-3}(3) + (5 + 3x)(-3x^{-4})$ {using equation 2 & 3}

$\Rightarrow \frac{dy}{dx} = 3x^{-3} - 15x^{-4} - 9x^{-3} = -15x^{-4} - 6x^{-3}$

Hence,

$$\frac{dy}{dx} = -x^{-4}(15 + 6x) \text{ ....ans}$$

25. Question

Differentiate the following functions with respect to x:

(ax + b)/(cx + d)

Answer

Let, y = (ax + b)/(cx + d)

We have to find dy/dx

As we can observe that y is a product of two functions say u and v where,

u = ax + b and v = cx + d

∴ y = uv

As we know that to find the derivative of product of two function we apply product rule of differentiation.

By product rule, we have –

$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$ ...equation 1

As, u = ax + b

$\therefore \frac{du}{dx} = a + 0 = a$ ...equation 2 $\{\because \frac{d}{dx}(x^n) = nx^{n-1}\}$

As, v = $\frac{1}{cx+d}$

As, $\frac{d}{dx}\left(\frac{1}{x}\right) = -\frac{1}{x^2}$ & $\frac{d}{dx}(f(ax + b)) = a f'(ax + b)$

$\therefore \frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{cx+d}\right) = -\frac{c}{(cx+d)^2}$ ...equation 3

∴ from equation 1, we can find dy/dx

$\therefore \frac{dy}{dx} = (ax + b)\frac{dv}{dx} + \left(\frac{1}{cx+d}\right)\frac{du}{dx}$

$\Rightarrow \frac{dy}{dx} = (ax + b)\left(\frac{-c}{(cx+d)^2}\right) + \left(\frac{1}{cx+d}\right)(a)$ {using equation 2 & 3}

$\Rightarrow \frac{dy}{dx} = \frac{-acx-bc+a(cx+d)}{(cx+d)^2} = \frac{ad-bc}{(cx+d)^2}$

Hence,

$$\frac{dy}{dx} = \frac{ad - bc}{(cx + d)^2} \text{ ....ans}$$

26. Question

Differentiate the following functions with respect to x:

$(ax + b)^n(cx + d)^m$

Answer

Let, $y = (ax + b)^n(cx + d)^m$

We have to find dy/dx

As we can observe that y is a product of two functions say u and v where,

$u = (ax + b)^n$ and $v = (cx + d)^m$

$\therefore y = uv$

As we know that to find the derivative of product of two function we apply product rule of differentiation.

By product rule, we have –

$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$ ...equation 1

As, $u = (ax + b)^n$

As, $\frac{d}{dx}(x^n) = nx^{n-1}$ & $\frac{d}{dx}(f(ax + b)) = a f'(ax + b)$

$\therefore \frac{du}{dx} = n(ax + b)^{n-1}$ ...equation 2

As, $v = (cx + d)^m$

As, $\frac{d}{dx}(x^n) = nx^{n-1}$ & $\frac{d}{dx}(f(ax + b)) = a f'(ax + b)$

$\therefore \frac{dv}{dx} = \frac{d}{dx}\{(cx + d)^m\} = m(cx + d)^{m-1}$ ...equation 3

$\therefore$ from equation 1, we can find dy/dx

$\therefore \frac{dy}{dx} = (ax + b)^n\frac{dv}{dx} + (cx + d)^m\frac{du}{dx}$

{using equation 2 & 3}

$\Rightarrow \frac{dy}{dx} = (ax + b)^n(m(cx + d)^{m-1}) + (cx + d)^m(n(ax + b)^{n-1})$

$\Rightarrow \frac{dy}{dx} = m(ax + b)^n(cx + d)^{m-1} + n(cx + d)^m(ax + b)^{n-1}$

Hence,

$$\frac{dy}{dx} = m(ax + b)^n(cx + d)^{m-1} + n(cx + d)^m(ax + b)^{n-1} \text{ ....ans}$$

27. Question

Differentiate in two ways, using product rule and otherwise, the function

(1 + 2tan x)(5 + 4 cos x). Verify that the answers are the same.

Answer

Let, y = (1 + 2 tan x)(5 + 4 cos x)

$\Rightarrow$ y = 5 + 4 cos x + 10 tan x + 8 tan x cos x

$\Rightarrow$ y = 5 + 4 cos x + 10 tan x + 8 sin x {$\because$ tan x cos x = sin x}

Differentiating y w.r.t x –

$$\frac{dy}{dx} = \frac{d}{dx}(5 + 4\cos x + 10\tan x + 8\sin x)$$

Using algebra of derivatives, we have –

$$\frac{dy}{dx} = \frac{d}{dx}(5) + 4\frac{d}{dx}(\cos x) + 10\frac{d}{dx}(\tan x) + 8\frac{d}{dx}(\sin x)$$

Use formula of derivative of above function to get the result.

$\Rightarrow \frac{dy}{dx} = 0 + 4(-\sin x) + 10\sec^2 x + 8\cos x$

$\therefore \frac{dy}{dx} = -4\sin x + 8\cos x + 10\sec^2 x$ ...equation 1

<u>Derivative using product rule –</u>

We have to find dy/dx

As we can observe that y is a product of two functions say u and v where,

u = (1 + 2tan x) and v = (5 + 4cos x)

∴ y = uv

As we know that to find the derivative of product of two function we apply product rule of differentiation.

By product rule, we have –

$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$ ...equation 2

As, u = (1 + 2tan x)

$\therefore \frac{du}{dx} = \frac{d}{dx}(1 + 2\tan x)$

$\Rightarrow \frac{du}{dx} = \frac{d}{dx}(1) + 2\frac{d}{dx}(\tan x) = 0 + 2\sec^2 x$

$\Rightarrow \frac{du}{dx} = 2\sec^2 x$ .....equation 3 $\{\because \frac{d}{dx}(\tan x) = \sec^2 x\}$

As, v = 5 + 4cos x

$\therefore \frac{dv}{dx} = \frac{d}{dx}(5 + 4\cos x)$

$\Rightarrow \frac{dv}{dx} = \frac{d}{dx}(5) + 4\frac{d}{dx}(\cos x) = 0 - 4\sin x$

$\Rightarrow \frac{dv}{dx} = -4\sin x$ ...equation 4 $\{\because \frac{d}{dx}(\cos x) = -\sin x\}$

∴ from equation 2, we can find dy/dx

$\therefore \frac{dy}{dx} = (1 + 2\tan x)\frac{dv}{dx} + (5 + 4\cos x)\frac{du}{dx}$

using equation 3 & 4, we get –

$\Rightarrow \frac{dy}{dx} = (1 + 2\tan x)(-4\sin x) + (5 + 4\cos x)(2\sec^2 x)$

$\Rightarrow \frac{dy}{dx} = -4\sin x + 8\tan x \times \sin x + 10\sec^2 x + 8\cos x \times \sec^2 x$

∵ sin x = tan x cos x , so we get –

$\Rightarrow \frac{dy}{dx} = 4\cos x + 8\frac{\sin^2 x}{\cos x} + 10\sec^2 x - 8\frac{1}{\cos x}$

$\Rightarrow \frac{dy}{dx} = 4\cos x + 10\sec^2 x - 8\sec x(1 - \sin^2 x)$

$\Rightarrow \frac{dy}{dx} = 4\cos x + 10\sec^2 x - 8\sec x\cos^2 x$ [∵ 1 – $\sin^2 x = \cos^2 x$]

$\therefore \frac{dy}{dx} = 4\cos x + 10\sec^2 x - 8\cos x$

Hence,

$\frac{dy}{dx} = 4\cos x + 10\sec^2 x - 8\cos x$ ....equation 5

Clearly from equation 1 and 5 we observed that both equations gave identical results.

Hence, Results are verified

28 A. Question

Differentiate each of the following functions by the product by the product rule and the other method and verify that answer from both the methods is the same.

$(3x^2 + 2)^2$

Answer

Let, $y = (3x^2 + 2)^2 = (3x^2 + 2)(3x^2 + 2)$

$\Rightarrow y = 9x^4 + 6x^2 + 6x^2 + 4$

$\Rightarrow y = 9x^4 + 12x^2 + 4$

Differentiating y w.r.t x –

$$\frac{dy}{dx} = \frac{d}{dx}(9x^4 + 12x^2 + 4)$$

Using algebra of derivatives, we have –

$$\frac{dy}{dx} = \frac{d}{dx}(9x^4) + 12\frac{d}{dx}(x^2) + \frac{d}{dx}(4)$$

Use formula of derivative of above function to get the result.

$\Rightarrow \frac{dy}{dx} = 9(4x^{4-1}) + 12(2x^{2-1}) + 0 \ \{\because \frac{d}{dx}(x^n) = nx^{n-1}\}$

$\therefore \frac{dy}{dx} = 36x^3 + 24x$ ...equation 1

<u>Derivative using product rule –</u>

We have to find dy/dx

As we can observe that y is a product of two functions say u and v where,

$u = (3x^2 + 2)$ and $v = (3x^2 + 2)$

$\therefore y = uv$

As we know that to find the derivative of product of two function we apply product rule of differentiation.

By product rule, we have –

$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$ ...equation 2

As, $u = (3x^2 + 2)$

$\therefore \frac{du}{dx} = \frac{d}{dx}(3x^2 + 2)$

$\Rightarrow \frac{du}{dx} = \frac{d}{dx}(3x^2) + 2\frac{d}{dx}(1) = 6x$

$\Rightarrow \frac{du}{dx} = 6x$ .....equation 3 $\{\because \frac{d}{dx}(x^n) = nx^{n-1}\}$

As, $v = (3x^2 + 2)$

$\therefore \frac{dv}{dx} = \frac{d}{dx}(3x^2 + 2)$

$\Rightarrow \frac{dv}{dx} = \frac{d}{dx}(3x^2) + 2\frac{d}{dx}(1) = 6x$

$\Rightarrow \frac{dv}{dx} = 6x$ ...equation 4 $\{\because \frac{d}{dx}(x^n) = nx^{n-1}\}$

∴ from equation 2, we can find dy/dx

$\therefore \frac{dy}{dx} = (3x^2 + 2)\frac{dv}{dx} + (3x^2 + 2)\frac{du}{dx}$

using equation 3 & 4, we get –

$\Rightarrow \frac{dy}{dx} = (3x^2 + 2)(6x) + (3x^2 + 2)(6x)$

$\Rightarrow \frac{dy}{dx} = 18x^3 + 12x + 18x^3 + 12x = 36x^3 + 24x$

Hence,

$\frac{dy}{dx} = 36x^3 + 24x$ ....equation 5

Clearly from equation 1 and 5 we observed that both equations gave identical results.

Hence, Results are verified

28 B. Question

Differentiate each of the following functions by the product by the product rule and the other method and verify that answer from both the methods is the same.

(x + 2)(x + 3)

Answer

Let, y = (x + 2)(x + 3)

$\Rightarrow y = x^2 + 5x + 6$

Differentiating y w.r.t x –

$$\frac{dy}{dx} = \frac{d}{dx}(x^2 + 5x + 6)$$

Using algebra of derivatives, we have –

$$\frac{dy}{dx} = \frac{d}{dx}(x^2) + 5\frac{d}{dx}(x) + \frac{d}{dx}(6)$$

Use formula of derivative of above function to get the result.

$\Rightarrow \frac{dy}{dx} = (2x^{2-1}) + 5(x^{1-1}) + 0 \{\because \frac{d}{dx}(x^n) = nx^{n-1}\}$

$\therefore \frac{dy}{dx} = 2x + 5$ ...equation 1

<u>Derivative using product rule –</u>

We have to find dy/dx

As we can observe that y is a product of two functions say u and v where,

u = (x + 2) and v = (x + 3)

∴ y = uv

As we know that to find the derivative of product of two function we apply product rule of differentiation.

By product rule, we have –

$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$...equation 2

As, u = (x + 2)

$\therefore \frac{du}{dx} = \frac{d}{dx}(x + 2)$

$\Rightarrow \frac{du}{dx} = \frac{d}{dx}(x) + \frac{d}{dx}(2) = 1$

$\Rightarrow \frac{du}{dx} = 1$ .....equation 3 $\{\because \frac{d}{dx}(x^n) = nx^{n-1}\}$

As, v = (x + 3)

$\therefore \frac{dv}{dx} = \frac{d}{dx}(x + 3)$

$\Rightarrow \frac{dv}{dx} = \frac{d}{dx}(x) + \frac{d}{dx}(3) = 1$

$\Rightarrow \frac{dv}{dx} = 1$ ...equation 4 $\{\because \frac{d}{dx}(x^n) = nx^{n-1}\}$

∴ from equation 2, we can find dy/dx

$\therefore \frac{dy}{dx} = (x + 2)\frac{dv}{dx} + (x + 3)\frac{du}{dx}$

using equation 3 & 4, we get –

$\Rightarrow \frac{dy}{dx} = (x + 2)(1) + (x + 3)(1)$

$\Rightarrow \frac{dy}{dx} = x + 2 + x + 3 = 2x + 5$

Hence,

$\frac{dy}{dx} = 2x + 5$ ....equation 5

Clearly from equation 1 and 5 we observed that both equations gave identical results.

Hence, Results are verified

28 C. Question

Differentiate each of the following functions by the product by the product rule and the other method and verify that answer from both the methods is the same.

(3 sec x – 4 cosec x) (– 2 sin x + 5 cos x)

Answer

Let, y = (3 sec x – 4 cosec x)(– 2 sin x + 5 cos x)

⇒ y = – 6 sec x sin x + 15 sec x cos x + 8 sin x cosec x – 20cosec x cos x

⇒ y = – 6tan x + 15 + 8 – 20 cot x {∵ tan x cos x = sin x}

⇒ y = – 6 tan x – 20 cot x + 23

Differentiating y w.r.t x –

$$\frac{dy}{dx} = \frac{d}{dx}(-6\tan x - 20\cot x + 23)$$

Using algebra of derivatives, we have –

$$\frac{dy}{dx} = (-6)\frac{d}{dx}(\tan x) - 20\frac{d}{dx}(\cot x) + \frac{d}{dx}(23)$$

Use formula of derivative of above function to get the result.

$\Rightarrow \frac{dy}{dx} = -6\sec^2 x - 20\left(-\operatorname{cosec}^2 x\right) + 0$

$\therefore \frac{dy}{dx} = 20\operatorname{cosec}^2 x - 6\sec^2 x$ ...equation 1

<u>Derivative using product rule –</u>

We have to find dy/dx

As we can observe that y is a product of two functions say u and v where,

u = (3 sec x – 4 cosec x) and v = (– 2 sin x + 5 cos x)

∴ y = uv

As we know that to find the derivative of product of two function we apply product rule of differentiation.

By product rule, we have –

$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$ ...equation 2

As, u = (3 sec x – 4 cosec x)

$\therefore \frac{du}{dx} = \frac{d}{dx}(3\sec x - 4\operatorname{cosec} x)$

Use the formula: $\frac{d}{dx}(\sec x) = \sec x \tan x$ & $\frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x \cot x$

$\Rightarrow \frac{du}{dx} = 3\frac{d}{dx}(\sec x) - 4\frac{d}{dx}(\operatorname{cosec} x) = 3\sec x\tan x - (-4\operatorname{cosec}\cot x)$

$\Rightarrow \frac{du}{dx} = 3\sec x\tan x + 4\operatorname{cosec} x\cot x$ .....equation 3

As, v = – 2 sin x + 5 cos x

$\therefore \frac{dv}{dx} = \frac{d}{dx}(-2\sin x + 5\cos x)$

$\Rightarrow \frac{dv}{dx} = -2\frac{d}{dx}(\sin x) + 5\frac{d}{dx}(\cos x) = -2\cos x - 5\sin x$

$\Rightarrow \frac{dv}{dx} = -2\cos x - 5\sin x$ ...equation 4 $\{\because \frac{d}{dx}(\cos x) = -\sin x\}$

∴ from equation 2, we can find dy/dx

$\therefore \frac{dy}{dx} = (1 + 2\tan x)\frac{dv}{dx} + (-2\sin x + 5\cos x)\frac{du}{dx}$

using equation 3 & 4, we get –

$\Rightarrow \frac{dy}{dx} = (3\sec x - 4\operatorname{cosec} x)(-2\cos x - 5\sin x) + (-2\sin x + 5\cos x)(3\sec x\tan x + 4\operatorname{cosec} x\cot x)$

∵ sin x = tan x cos x , so we get –

$\Rightarrow \frac{dy}{dx} = (-15\tan x + 8\cot x + 14) + (-6\tan^2 x - 8\cot x + 15\tan x + 20\cot^2 x)$

$\Rightarrow \frac{dy}{dx} = 14 - 6\tan^2 x + 20\cot^2 x$

$\Rightarrow \frac{dy}{dx} = 20 - 6 - 6\tan^2 x + 20\cot^2 x$ = 20(1 + $\cot^2$ x) – 6(1 + $\tan^2$ x)

$\therefore \frac{dy}{dx} = 20\operatorname{cosec}^2 x - 6\sec^2 x$ [∵ 1 + $\tan^2$ x = $\sec^2$ x & 1 + $\cot^2$ x = cosec^2 x]

$\therefore \frac{dy}{dx} = 20\operatorname{cosec}^2 x - 6\sec^2 x$

Hence,

$\frac{dy}{dx} = 20\operatorname{cosec}^2 x - 6\sec^2 x$ ....equation 5

Clearly from equation 1 and 5 we observed that both equations gave identical results.

Hence, Results are verified

Exercise 30.5

1. Question

Differentiate the following functions with respect to x:

$$\frac{x^2 + 1}{x + 1}$$

Answer

Let, $y = \frac{x^2 + 1}{x + 1}$

We have to find dy/dx

As we can observe that y is a fraction of two functions say u and v where,

$u = x^2 + 1$ and $v = x + 1$

$\therefore y = u/v$

As we know that to find the derivative of fraction of two function we apply quotient rule of differentiation.

By quotient rule, we have –

$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$ ...equation 1

As, $u = x^2 + 1$

$\therefore \frac{du}{dx} = 2x^{2-1} + 0 = 2x$ ...equation 2 $\{\because \frac{d}{dx}(x^n) = nx^{n-1}\}$

As, $v = x + 1$

$\therefore \frac{dv}{dx} = \frac{d}{dx}(x + 1) = 1$ ...equation 3 $\{\because \frac{d}{dx}(x^n) = nx^{n-1}\}$

$\therefore$ from equation 1, we can find dy/dx

$\therefore \frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

$\Rightarrow \frac{dy}{dx} = \frac{(x+1)(2x) - (x^2+1)(1)}{(x+1)^2}$ {using equation 2 and 3}

$\Rightarrow \frac{dy}{dx} = \frac{2x^2 + 2x - x^2 - 1}{(x+1)^2}$

$\Rightarrow \frac{dy}{dx} = \frac{x^2 + 2x - 1}{(x+1)^2}$

Hence,

$$\frac{dy}{dx} = \frac{x^2 + 2x - 1}{(x + 1)^2} \text{ ....ans}$$

2. Question

Differentiate the following functions with respect to x:

$$\frac{2x-1}{x^2+1}$$

Answer

Let, $y = \frac{2x-1}{x^2+1}$

We have to find dy/dx

As we can observe that y is a fraction of two functions say u and v where,

u = 2x – 1 and v = x^2 + 1

∴ y = u/v

As we know that to find the derivative of fraction of two function we apply quotient rule of differentiation.

By quotient rule, we have –

$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$ ...equation 1

As, u = 2x – 1

$\therefore \frac{du}{dx} = 2x^{1-1} - 0 = 2$ ...equation 2 $\{\because \frac{d}{dx}(x^n) = nx^{n-1}\}$

As, v = x^2 + 1

$\therefore \frac{dv}{dx} = \frac{d}{dx}(x^2 + 1) = 2x$ ...equation 3 $\{\because \frac{d}{dx}(x^n) = nx^{n-1}\}$

∴ from equation 1, we can find dy/dx

$\therefore \frac{dy}{dx} = \frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$

$\Rightarrow \frac{dy}{dx} = \frac{(x^2+1)(2)-(2x-1)(2x)}{(x^2+1)^2}$ {using equation 2 and 3}

$\Rightarrow \frac{dy}{dx} = \frac{2x^2+2-4x^2+2x}{(x+1)^2}$

$\Rightarrow \frac{dy}{dx} = \frac{-2x^2+2x+2}{(x+1)^2}$

Hence,

$$\frac{dy}{dx} = \frac{-2x^2 + 2x + 2}{(x + 1)^2} \text{ ....ans}$$

3. Question

Differentiate the following functions with respect to x:

$$\frac{x+e^x}{1+\log x}$$

Answer

Let, $y = \frac{x+e^x}{1+\log x}$

We have to find dy/dx

As we can observe that y is a fraction of two functions say u and v where,

$u = x + e^x$ and $v = 1 + \log x$

$\therefore y = u/v$

As we know that to find the derivative of fraction of two function we apply quotient rule of differentiation.

By quotient rule, we have –

$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$ ...equation 1

As, $u = x + e^x$

$\therefore \frac{du}{dx} = \frac{d}{dx}(x + e^x)$

$\because \frac{d}{dx}(x^n) = nx^{n-1}$ & $\frac{d}{dx}(e^x) = e^x$, so we get –

$\Rightarrow \frac{du}{dx} = \frac{d}{dx}(x) + \frac{d}{dx}(e^x) = 1 + e^x$ ...equation 2

As, $v = 1 + \log x$

$\therefore \frac{dv}{dx} = \frac{d}{dx}(\log x + 1) = \frac{d}{dx}(1) + \frac{d}{dx}(\log x)$

$\Rightarrow \frac{dv}{dx} = 0 + \frac{1}{x} = \frac{1}{x}$ ...equation 3 $\{\because \frac{d}{dx}(\log x) = \frac{1}{x}\}$

$\therefore$ from equation 1, we can find dy/dx

$\therefore \frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

$\Rightarrow \frac{dy}{dx} = \frac{(1 + \log x)(1 + e^x) - (x + e^x)\left(\frac{1}{x}\right)}{(\log x + 1)^2}$ {using equation 2 and 3}

$\Rightarrow \frac{dy}{dx} = \frac{1 + e^x + \log x + e^x \log x - 1 - \frac{e^x}{x}}{(\log x + 1)^2} = \frac{x\log x(1 + e^x) + e^x(x-1)}{x(\log x + 1)^2}$

$\Rightarrow \frac{dy}{dx} = \frac{x\log x(1 + e^x) + e^x(x-1)}{x(\log x + 1)^2}$

Hence,

$$\frac{dy}{dx} = \frac{x\log x(1 + e^x) + e^x(x-1)}{x(\log x + 1)^2} \text{ .... ans}$$

4. Question

Differentiate the following functions with respect to x:

$$\frac{e^x - \tan x}{\cot x - x^n}$$

Answer

Let, $y = \frac{e^x - \tan x}{\cot x - x^n}$

We have to find dy/dx

As we can observe that y is a fraction of two functions say u and v where,

$u = e^x - \tan x$ and $v = \cot x - x^n$

$\therefore y = u/v$

As we know that to find the derivative of fraction of two function we apply quotient rule of differentiation.

By quotient rule, we have –

$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$ ...equation 1

As, u = e^x – tan x

$\therefore \frac{du}{dx} = \frac{d}{dx}(e^x - \tan x)$

$\because \frac{d}{dx}(\tan x) = \sec^2 x$ & $\frac{d}{dx}(e^x) = e^x$, so we get –

$\Rightarrow \frac{du}{dx} = -\frac{d}{dx}(\tan x) + \frac{d}{dx}(e^x) = \sec^2 x + e^x$ ...equation 2

As, v = cot x – x^n

$\therefore \frac{dv}{dx} = \frac{d}{dx}(\cot x - x^n) = \frac{d}{dx}(\cot x) - \frac{d}{dx}(x^n)$

$\because \frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$ & $\frac{d}{dx}(x^n) = nx^{n-1}$, so we get –

$\Rightarrow \frac{dv}{dx} = -\operatorname{cosec}^2 x - nx^{n-1}$ ...equation 3

∴ from equation 1, we can find dy/dx

$\therefore \frac{dy}{dx} = \frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$

using equation 2 and 3, we get –

$\Rightarrow \frac{dy}{dx} = \frac{(\cot x - x^n)(\sec^2 x + e^x) - (e^x - \tan x)(-\operatorname{cosec}^2 x - nx^{n-1})}{(\cot x - x^n)^2}$

$\Rightarrow \frac{dy}{dx} = \frac{(\cot x - x^n)(\sec^2 x + e^x) + (e^x - \tan x)(+\operatorname{cosec}^2 x + nx^{n-1})}{(\cot x - x^n)^2}$

Hence,

$$\frac{dy}{dx} = \frac{(\cot x - x^n)(\sec^2 x + e^x) + (e^x - \tan x)(+\operatorname{cosec}^2 x + nx^{n-1})}{(\cot x - x^n)^2} \text{ .... ans}$$

5. Question

Differentiate the following functions with respect to x:

$$\frac{ax^2 + bx + c}{px^2 + qx + r}$$

Answer

Let, y = $\frac{ax^2 + bx + c}{px^2 + qx + r}$

We have to find dy/dx

As we can observe that y is a fraction of two functions say u and v where,

u = ax^2 + bx + c and v = px^2 + qx + r

∴ y = u/v

As we know that to find the derivative of fraction of two function we apply quotient rule of differentiation.

By quotient rule, we have –

$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$ ...equation 1

As, u = $ax^2 + bx + c$

$\therefore \frac{du}{dx} = 2ax + b$ ...equation 2 $\{\because \frac{d}{dx}(x^n) = nx^{n-1}\}$

As, v = $px^2 + qx + r$

$\because \frac{d}{dx}(x^n) = nx^{n-1}$, so we get –

$\therefore \frac{dv}{dx} = \frac{d}{dx}(px^2 + qx + r) = 2px + q$ ...equation 3

∴ from equation 1, we can find dy/dx

$\therefore \frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

$\Rightarrow \frac{dy}{dx} = \frac{(px^2 + qx + r)(2ax + b) - (ax^2 + bx + c)(2px + q)}{(px^2 + qx + r)^2}$ {using equation 2 and 3}

$\Rightarrow \frac{dy}{dx} = \frac{2apx^3 + bpx^2 + 2aqx^2 + bqx + 2arx + br - 2apx^3 - aqx^2 - 2bpx^2 - bqx - 2pcx - qc}{(px^2 + qx + r)^2}$

$\Rightarrow \frac{dy}{dx} = \frac{aqx^2 - bpx^2 + 2arx + br - 2pcx - qc}{(px^2 + qx + r)^2}$

$\Rightarrow \frac{dy}{dx} = \frac{x^2(aq - bp) + 2x(ar - pc) + br - qc}{(px^2 + qx + r)^2}$

Hence,

$$\frac{dy}{dx} = \frac{x^2(aq - bp) + 2x(ar - pc) + br - qc}{(px^2 + qx + r)^2} \ldots\text{ans}$$

6. Question

Differentiate the following functions with respect to x:

$$\frac{x}{1 + \tan x}$$

Answer

Let, y = $\frac{x}{1 + \tan x}$

We have to find dy/dx

As we can observe that y is a fraction of two functions say u and v where,

u = x and v = 1 + tan x

∴ y = u/v

As we know that to find the derivative of fraction of two function we apply quotient rule of differentiation.

By quotient rule, we have –

$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$ ...equation 1

As, u = x

$\{\because \frac{d}{dx}(x^n) = nx^{n-1}\}$

$\therefore \frac{du}{dx} = 1$ ...equation 2

As, v = 1 + tan x

$\because \frac{d}{dx}(\tan x) = \sec^2 x$, so we get –

$\therefore \frac{dv}{dx} = \frac{d}{dx}(1 + \tan x) = 0 + \sec^2 x = \sec^2 x$ ...equation 3

∴ from equation 1, we can find dy/dx

$\therefore \frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

$\Rightarrow \frac{dy}{dx} = \frac{(1 + \tan x)(1) - (x)(\sec^2 x)}{(1 + \tan x)^2}$ {using equation 2 and 3}

$\Rightarrow \frac{dy}{dx} = \frac{1 + \tan x - x\sec^2 x}{(1 + \tan x)^2}$

Hence,

$$\frac{dy}{dx} = \frac{1 + \tan x - x\sec^2 x}{(1 + \tan x)^2} \ldots\text{. ans}$$

7. Question

Differentiate the following functions with respect to x:

$$\frac{1}{ax^2 + bx + c}$$

Answer

Let, $y = \frac{1}{ax^2 + bx + c}$

We have to find dy/dx

As we can observe that y is a fraction of two functions say u and v where,

$u = 1$ and $v = ax^2 + bx + c$

∴ y = u/v

As we know that to find the derivative of fraction of two function we apply quotient rule of differentiation.

By quotient rule, we have –

$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$ ...equation 1

As, u = 1

$\therefore \frac{du}{dx} = 0$ ...equation 2 {$\because \frac{d}{dx}(x^n) = nx^{n-1}$ }

As, $v = ax^2 + bx + c$

$\because \frac{d}{dx}(x^n) = nx^{n-1}$, so we get –

$\therefore \frac{dv}{dx} = \frac{d}{dx}(ax^2 + bx + c) = 2ax + b$ ...equation 3

∴ from equation 1, we can find dy/dx

$\therefore \frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

$\Rightarrow \frac{dy}{dx} = \frac{(ax^2 + bx + c)(0) - (1)(2ax + b)}{(ax^2 + bx + c)^2}$ {using equation 2 and 3}

$\Rightarrow \frac{dy}{dx} = \frac{-(2ax + b)}{(ax^2 + bx + c)^2}$

Hence,

$$\frac{dy}{dx} = -\frac{(2ax + b)}{(ax^2 + bx + c)^2} \text{ .... ans}$$

8. Question

Differentiate the following functions with respect to x:

$$\frac{e^x}{1 + x^2}$$

Answer

Let, $y = \frac{e^x}{x^2 + 1}$

We have to find dy/dx

As we can observe that y is a fraction of two functions say u and v where,

$u = e^x$ and $v = x^2 + 1$

$\therefore y = u/v$

As we know that to find the derivative of fraction of two function we apply quotient rule of differentiation.

By quotient rule, we have –

$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$ ...equation 1

As, $u = e^x$

$\therefore \frac{du}{dx} = \frac{d}{dx}(e^x) = e^x$ ...equation 2 $\{\because \frac{d}{dx}(e^x) = e^x\}$

As, $v = x^2 + 1$

$\therefore \frac{dv}{dx} = \frac{d}{dx}(x^2 + 1) = 2x$ ...equation 3 $\{\because \frac{d}{dx}(x^n) = nx^{n-1}\}$

$\therefore$ from equation 1, we can find dy/dx

$\therefore \frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

$\Rightarrow \frac{dy}{dx} = \frac{(x^2 + 1)(e^x) - (2x-1)(e^x)}{(x^2 + 1)^2}$ {using equation 2 and 3}

$\Rightarrow \frac{dy}{dx} = \frac{e^x(x^2 + 1 - 2x + 1)}{(x + 1)^2}$

$\Rightarrow \frac{dy}{dx} = \frac{e^x(x^2 - 2x + 2)}{(x + 1)^2}$

Hence,

$$\frac{dy}{dx} = \frac{e^x(x^2 - 2x + 2)}{(x + 1)^2} \text{ .... ans}$$

9. Question

Differentiate the following functions with respect to x:

$$\frac{e^x + \sin x}{1 + \log x}$$

Answer

Let, $y = \frac{e^x + \sin x}{1 + \log x}$

We have to find dy/dx

As we can observe that y is a fraction of two functions say u and v where,

u = e^x + sin x and v = 1 + log x

∴ y = u/v

As we know that to find the derivative of fraction of two function we apply quotient rule of differentiation.

By quotient rule, we have –

$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$ ...equation 1

As, u = sin x + e^x

$\therefore \frac{du}{dx} = \frac{d}{dx}(\sin x + e^x)$

$\because \frac{d}{dx}(\sin x) = \cos x \,\&\, \frac{d}{dx}(e^x) = e^x$, so we get –

$\Rightarrow \frac{du}{dx} = \frac{d}{dx}(\sin x) + \frac{d}{dx}(e^x) = \cos x + e^x$ ...equation 2

As, v = 1 + log x

$\therefore \frac{dv}{dx} = \frac{d}{dx}(\log x + 1) = \frac{d}{dx}(1) + \frac{d}{dx}(\log x)$

$\Rightarrow \frac{dv}{dx} = 0 + \frac{1}{x} = \frac{1}{x}$ ...equation 3 $\{\because \frac{d}{dx}(\log x) = \frac{1}{x}\}$

∴ from equation 1, we can find dy/dx

$\therefore \frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

$\Rightarrow \frac{dy}{dx} = \frac{(1 + \log x)(\cos x + e^x) - (\sin x + e^x)(\frac{1}{x})}{(\log x + 1)^2}$ {using equation 2 and 3}

$\Rightarrow \frac{dy}{dx} = \frac{x(1 + \log x)(\cos x + e^x) - (\sin x + e^x)}{x(\log x + 1)^2}$

Hence,

$$\frac{dy}{dx} = \frac{x(1 + \log x)(\cos x + e^x) - (\sin x + e^x)}{x(\log x + 1)^2} \text{ ....ans}$$

10. Question

Differentiate the following functions with respect to x:

$$\frac{x\tan x}{\sec x + \tan x}$$

Answer

Let, $y = \frac{x \tan x}{\sec x + \tan x}$

We have to find dy/dx

As we can observe that y is a fraction of two functions say u and v where,

u = x tan x and v = sec x + tan x

∴ y = u/v

As we know that to find the derivative of fraction of two function we apply quotient rule of differentiation.

By quotient rule, we have –

$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$ ...equation 1

As, u = x tan x

∵ u is the product of two function x and tan x, so we will be applying product rule of differentiation –

$\therefore \frac{du}{dx} = \frac{d}{dx}(x \tan x)$

$\Rightarrow \frac{du}{dx} = x\frac{d}{dx}(\tan x) + \tan x \frac{d}{dx}(x)$ [using product rule]

$\because \frac{d}{dx}(\tan x) = \sec^2 x$ & $\frac{d}{dx}(x^n) = nx^{n-1}$, So we get –

$\Rightarrow \frac{du}{dx} = x\sec^2 x + \tan x$ ...equation 2

As, v = sec x + tan x

$\because \frac{d}{dx}(\tan x) = \sec^2 x$ & $\frac{d}{dx}(\sec x) = \sec x \tan x$, so we get –

$\therefore \frac{dv}{dx} = \frac{d}{dx}(\sec x + \tan x) = \sec x \tan x + \sec^2 x$ ...equation 3

∴ from equation 1, we can find dy/dx

$\therefore \frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

using equation 2 and 3, we get –

$\Rightarrow \frac{dy}{dx} = \frac{(\sec x + \tan x)(x\sec^2 x + \tan x) - (x\tan x)(\sec^2 x + \sec x \tan x)}{(\sec x + \tan x)^2}$

$\Rightarrow \frac{dy}{dx} = \frac{(\sec x + \tan x)(x\sec^2 x + \tan x) - (x\tan x)(\sec x + \tan x)\sec x}{(\sec x + \tan x)^2}$

$\Rightarrow \frac{dy}{dx} = \frac{(\sec x + \tan x)(x\sec^2 x + \tan x - x\tan x \sec x)}{(\sec x + \tan x)^2}$

$\Rightarrow \frac{dy}{dx} = \frac{(x\sec^2 x + \tan x - x\tan x \sec x)}{(\sec x + \tan x)}$

Hence,

$$\frac{dy}{dx} = \frac{(x\sec^2 x + \tan x - x\tan x \sec x)}{(\sec x + \tan x)} \text{ ....ans}$$

11. Question

Differentiate the following functions with respect to x:

$$\frac{x \sin x}{1 + \cos x}$$

Answer

Let, $y = \frac{x \sin x}{1 + \cos x}$

We have to find dy/dx

As we can observe that y is a fraction of two functions say u and v where,

u = x sin x and v = 1 + cos x

∴ y = u/v

As we know that to find the derivative of fraction of two function we apply quotient rule of differentiation.

By quotient rule, we have –

$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$ ...equation 1

As, u = x sin x

∵ u is the product of two function x and tan x, so we will be applying product rule of differentiation –

$\therefore \frac{du}{dx} = \frac{d}{dx}(x \sin x)$

$\Rightarrow \frac{du}{dx} = x\frac{d}{dx}(\sin x) + \sin x \frac{d}{dx}(x)$ [using product rule]

$\because \frac{d}{dx}(\sin x) = \cos x$ & $\frac{d}{dx}(x^n) = nx^{n-1}$, So we get –

$\Rightarrow \frac{du}{dx} = x\cos x + \sin x$ ...equation 2

As, v = 1 + cos x

$\because \frac{d}{dx}(\cos x) = -\sin x$, so we get –

$\therefore \frac{dv}{dx} = \frac{d}{dx}(1 + \cos x) = 0 - \sin x = -\sin x$ ...equation 3

∴ from equation 1, we can find dy/dx

$\therefore \frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

using equation 2 and 3, we get –

$\Rightarrow \frac{dy}{dx} = \frac{(1 + \cos x)(x\cos x + \sin x) - (x \sin x)(-\sin x)}{(1 + \cos x)^2}$

$\Rightarrow \frac{dy}{dx} = \frac{x\cos x + \sin x + \cos x \sin x + x\cos^2 x + x\sin^2 x}{(1 + \cos x)^2}$

$\Rightarrow \frac{dy}{dx} = \frac{x\cos x + \sin x + \cos x \sin x + x(\cos^2 x + \sin^2 x)}{(1 + \cos x)^2} = \frac{\cos x(x + \sin x) + (\sin x + x)}{(1 + \cos x)^2}$

$\Rightarrow \frac{dy}{dx} = \frac{(\cos x + 1)(x + \sin x)}{(1 + \cos x)^2}$

$\Rightarrow \frac{dy}{dx} = \frac{x + \sin x}{(1 + \cos x)}$

Hence,

$\frac{dy}{dx} = \frac{x + \sin x}{(1 + \cos x)}$ .... ans

12. Question

Differentiate the following functions with respect to x:

$\frac{2^x \cot x}{\sqrt{x}}$

Answer

Let, $y = \frac{2^x \cot x}{\sqrt{x}}$

We have to find dy/dx

As we can observe that y is a fraction of two functions say u and v where,

$u = 2^x \cot x$ and $v = \sqrt{x}$

$\therefore y = u/v$

As we know that to find the derivative of fraction of two function we apply quotient rule of differentiation.

By quotient rule, we have –

$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$ ...equation 1

As, $u = 2^x \cot x$

$\because$ u is the product of two function x and tan x, so we will be applying product rule of differentiation –

$\therefore \frac{du}{dx} = \frac{d}{dx}(2^x \cot x)$

$\Rightarrow \frac{du}{dx} = 2^x \frac{d}{dx}(\cot x) + \cot x \frac{d}{dx}(2^x)$ [using product rule]

$\because \frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$ & $\frac{d}{dx}(a^x) = a^x \log_e a$, So we get –

$\Rightarrow \frac{du}{dx} = 2^x(-\operatorname{cosec}^2 x) + \cot x\,(2^x \log_e 2)$ ...equation 2

As, $v = \sqrt{x}$

$\because \frac{d}{dx}(x^n) = nx^{n-1}$, so we get –

$\therefore \frac{dv}{dx} = \frac{d}{dx}\left(x^{\frac{1}{2}}\right) = \frac{1}{2}x^{\frac{1}{2}-1} = \frac{1}{2\sqrt{x}}$ ...equation 3

$\therefore$ from equation 1, we can find dy/dx

$\therefore \frac{dy}{dx} = \frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$

using equation 2 and 3, we get –

$\Rightarrow \frac{dy}{dx} = \frac{(\sqrt{x})(-2^x \operatorname{cosec}^2 x + 2^x \cot x \log 2)-(2^x \cot x)(\frac{1}{2\sqrt{x}})}{(\sqrt{x})^2}$

$\Rightarrow \frac{dy}{dx} = \frac{(x)(-2^x \operatorname{cosec}^2 x + 2^x \cot x \log 2)-(2^x \cot x)}{2x\sqrt{x}}$

$\Rightarrow \frac{dy}{dx} = \frac{(2^x)(-x \operatorname{cosec}^2 x + x \cot x \log 2 - \cot x)}{2x\sqrt{x}}$

Hence,

$$\frac{dy}{dx} = \frac{(2^x)(-x \operatorname{cosec}^2 x + x \cot x \log 2 - \cot x)}{2x\sqrt{x}} \text{ .... ans}$$

13. Question

Differentiate the following functions with respect to x:

$$\frac{\sin x - x\cos x}{x\sin x + \cos x}$$

Answer

Let, $y = \frac{\sin x - x\cos x}{x\sin x + \cos x}$

We have to find dy/dx

As we can observe that y is a fraction of two functions say u and v where,

u = sin x – x cos x and v = x sin x + cos x

∴ y = u/v

As we know that to find the derivative of fraction of two function we apply quotient rule of differentiation.

By quotient rule, we have –

$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$ ...equation 1

u = – (x cos x – sin x)

$\therefore \frac{du}{dx} = -\frac{d}{dx}(x \cos x - \sin x)$

Using algebra of derivatives –

$\Rightarrow \frac{du}{dx} = -\frac{d}{dx}(x \cos x) + \frac{d}{dx}(\sin x)$

$\because \frac{d}{dx}(\sin x) = \cos x \;\&\; \frac{d}{dx}(\cos x) = -\sin x$

$\therefore \frac{du}{dx} = -x\frac{d}{dx}(\cos x) - \cos x \frac{d}{dx}(x) + \frac{d}{dx}(\sin x)$ {using product rule}

$\Rightarrow \frac{du}{dx} = x \sin x - \cos x + \cos x = x \sin x$ ...equation 2

As, v = x sin x + cos x

$\therefore \frac{dv}{dx} = \frac{d}{dx}(x \sin x + \cos x)$

Using algebra of derivatives –

$\Rightarrow \frac{dv}{dx} = \frac{d}{dx}(x \sin x) + \frac{d}{dx}(\cos x)$

$\because \frac{d}{dx}(\sin x) = \cos x \;\&\; \frac{d}{dx}(\cos x) = -\sin x$

$\therefore \frac{dv}{dx} = x\frac{d}{dx}(\sin x) + \sin x\frac{d}{dx}(x) + \frac{d}{dx}(\cos x)$ {using product rule}

$\Rightarrow \frac{dv}{dx} = x \cos x + \sin x - \sin x = x \cos x$ ...equation 3

∴ from equation 1, we can find dy/dx

$\therefore \frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

using equation 2 and 3, we get –

$\Rightarrow \frac{dy}{dx} = \frac{(x \sin x + \cos x)(x \sin x) - (\sin x - x \cos x)(x \cos x)}{(x \sin x + \cos x)^2}$

$\Rightarrow \frac{dy}{dx} = \frac{x^2 \sin^2 x + x \sin x \cos x - x \sin x \cos x + x^2 \cos^2 x}{(x \sin x + \cos x)^2}$

$\Rightarrow \frac{dy}{dx} = \frac{x^2 (\sin^2 x + \cos^2 x)}{(x \sin x + \cos x)^2} = \frac{x^2}{(x \sin x + \cos x)^2}$

Hence,

$\frac{dy}{dx} = \frac{x^2}{(x \sin x + \cos x)^2}$ ....ans

14. Question

Differentiate the following functions with respect to x:

$$\frac{x^2 - x + 1}{x^2 + x + 1}$$

Answer

Let, $y = \frac{x^2 - x + 1}{x^2 + x + 1}$

We have to find dy/dx

As we can observe that y is a fraction of two functions say u and v where,

$u = x^2 - x + 1$ and $v = x^2 + x + 1$

∴ y = u/v

As we know that to find the derivative of fraction of two function we apply quotient rule of differentiation.

By quotient rule, we have –

$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$ ...equation 1

As, $u = x^2 - x + 1$

$\therefore \frac{du}{dx} = 2x^{2-1} - 1 + 0 = 2x - 1$ ...equation 2 $\{\because \frac{d}{dx}(x^n) = nx^{n-1}\}$

As, $v = x^2 + x + 1$

$\therefore \frac{dv}{dx} = \frac{d}{dx}(x^2 + x + 1) = 2x + 1$ ...equation 3 $\{\because \frac{d}{dx}(x^n) = nx^{n-1}\}$

∴ from equation 1, we can find dy/dx

$\therefore \frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

$\Rightarrow \frac{dy}{dx} = \frac{(x^2 + x + 1)(2x-1) - (x^2 - x + 1)(2x + 1)}{(x^2 + x + 1)^2}$ {using equation 2 and 3}

$\Rightarrow \frac{dy}{dx} = \frac{2x^3 + 2x^2 + 2x - x^2 - x - 1 - 2x^3 + 2x^2 - 2x - x^2 + x - 1}{(x^2 + x + 1)^2}$

$\Rightarrow \frac{dy}{dx} = \frac{2x^2 - 2}{(x^2 + x + 1)^2}$

Hence,

$$\frac{dy}{dx} = \frac{2x^2 - 2}{(x^2 + x + 1)^2} \text{ .... ans}$$

15. Question

Differentiate the following functions with respect to x:

$$\frac{\sqrt{a} + \sqrt{x}}{\sqrt{a} - \sqrt{x}}$$

Answer

Let, $y = \frac{\sqrt{a} + \sqrt{x}}{\sqrt{a} - \sqrt{x}}$

We have to find dy/dx

As we can observe that y is a fraction of two functions say u and v where,

u = √a + √x and v = √a – √x

∴ y = u/v

As we know that to find the derivative of fraction of two function we apply quotient rule of differentiation.

By quotient rule, we have –

$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$ ...equation 1

As, u = √a + √x

$\because \frac{d}{dx}(x^n) = nx^{n-1}$, so we get –

$\therefore \frac{du}{dx} = \frac{d}{dx}\left(\sqrt{a} + x^{\frac{1}{2}}\right) = 0 + \frac{1}{2}x^{\frac{1}{2}-1} = \frac{1}{2\sqrt{x}}$ ...equation 2

As, v = √a – √x

$\because \frac{d}{dx}(x^n) = nx^{n-1}$, so we get –

$\therefore \frac{dv}{dx} = \frac{d}{dx}\left(\sqrt{a} - x^{\frac{1}{2}}\right) = 0 - \frac{1}{2}x^{\frac{1}{2}-1} = -\frac{1}{2\sqrt{x}}$ ...equation 3

∴ from equation 1, we can find dy/dx

$\therefore \frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

using equation 2 and 3, we get –

$\Rightarrow \frac{dy}{dx} = \frac{(\sqrt{a}-\sqrt{x})\left(\frac{1}{2\sqrt{x}}\right) - (\sqrt{a}+\sqrt{x})(-\frac{1}{2\sqrt{x}})}{(\sqrt{a}-\sqrt{x})^2}$

$\Rightarrow \frac{dy}{dx} = \frac{\frac{1}{2\sqrt{x}}\{(\sqrt{a}-\sqrt{x}) + (\sqrt{a}+\sqrt{x})\}}{(\sqrt{a}-\sqrt{x})^2}$

$\Rightarrow \frac{dy}{dx} = \frac{\frac{1}{2\sqrt{x}}\{2\sqrt{a}\}}{(\sqrt{a}-\sqrt{x})^2} = \frac{\sqrt{a}}{\sqrt{x}(\sqrt{a}-\sqrt{x})^2}$

Hence,

$$\frac{dy}{dx} = \frac{\sqrt{a}}{\sqrt{x}(\sqrt{a}-\sqrt{x})^2} \text{ .... ans}$$

16. Question

Differentiate the following functions with respect to x:

$$\frac{a + \sin x}{1 + a\sin x}$$

Answer

Let, $y = \frac{a + \sin x}{1 + a\sin x}$

We have to find dy/dx

As we can observe that y is a fraction of two functions say u and v where,

u = a + sin x and v = 1 + a sin x

∴ y = u/v

As we know that to find the derivative of fraction of two function we apply quotient rule of differentiation.

By quotient rule, we have –

$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$ ...equation 1

As, u = a + sin x

$\because \frac{d}{dx}(\sin x) = \cos x$, so we get –

$\therefore \frac{du}{dx} = \frac{d}{dx}(a + \sin x) = 0 + \cos x = \cos x$ ...equation 2

As, v = 1 + a sin x

$\because \frac{d}{dx}(\sin x) = \cos x$, so we get –

$\therefore \frac{dv}{dx} = \frac{d}{dx}(1 + a\sin x) = 0 + a\cos x = a\cos x$ ...equation 3

∴ from equation 1, we can find dy/dx

$\therefore \frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

using equation 2 and 3, we get –

$\Rightarrow \frac{dy}{dx} = \frac{(1 + a\sin x)(\cos x) - (a + \sin x)(a\cos x)}{(1 + a\sin x)^2}$

$\Rightarrow \frac{dy}{dx} = \frac{\cos x + a\sin x\cos x - a^2\cos x - a\sin x\cos x}{(1 + a\sin x)^2}$

$\Rightarrow \frac{dy}{dx} = \frac{\cos x - a^2\cos x}{(1 + a\sin x)^2}$

$\Rightarrow \frac{dy}{dx} = \frac{\cos x(1 - a^2)}{(1 + a\sin x)^2}$

Hence,

$$\frac{dy}{dx} = \frac{\cos x\,(1 - a^2)}{(1 + a\sin x)^2} \text{ ....ans}$$

17. Question

Differentiate the following functions with respect to x:

$$\frac{10^x}{\sin x}$$

Answer

Let, $y = \frac{10^x}{\sin x}$

We have to find dy/dx

As we can observe that y is a fraction of two functions say u and v where,

$u = 10^x$ and v = sin x

∴ y = u/v

As we know that to find the derivative of fraction of two function we apply quotient rule of differentiation.

By quotient rule, we have –

$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$ ...equation 1

As, $u = 10^x$

$\because \frac{d}{dx}(a^x) = a^x \log a$, so we get –

$\therefore \frac{du}{dx} = \frac{d}{dx}(10^x) = 10^x \log_e 10$ ...equation 2

As, v = sin x

$\because \frac{d}{dx}(\sin x) = \cos x$, so we get –

$\therefore \frac{dv}{dx} = \frac{d}{dx}(\sin x) = \cos x$ ...equation 3

∴ from equation 1, we can find dy/dx

$\therefore \frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

using equation 2 and 3, we get –

$\Rightarrow \frac{dy}{dx} = \frac{\sin x(10^x \log 10) - (10^x)(\cos x)}{\sin^2 x}$

$\Rightarrow \frac{dy}{dx} = \frac{10^x (\log 10 \sin x - \cos x)}{\sin^2 x}$

Hence,

$$\frac{dy}{dx} = \frac{10^x (\log 10 \sin x - \cos x)}{\sin^2 x} \text{ ....ans}$$

18. Question

Differentiate the following functions with respect to x:

$$\frac{1 + 3^x}{1 - 3^x}$$

Answer

Let, $y = \frac{1 + 3^x}{1 - 3^x}$

We have to find dy/dx

As we can observe that y is a fraction of two functions say u and v where,

$u = 1 + 3^x$ and $v = 1 - 3^x$

∴ y = u/v

As we know that to find the derivative of fraction of two function we apply quotient rule of differentiation.

By quotient rule, we have –

$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$ ...equation 1

As, $u = 1 + 3^x$

$\because \frac{d}{dx}(a^x) = a^x \log a$, so we get –

$\therefore \frac{du}{dx} = \frac{d}{dx}(1 + 3^x) = 3^x \log_e 3$ ...equation 2

As, $v = 1 - 3^x$

$\because \frac{d}{dx}(a^x) = a^x \log a$, so we get –

$\therefore \frac{dv}{dx} = \frac{d}{dx}(1 - 3^x) = -3^x \log_e 3$ ...equation 3

∴ from equation 1, we can find dy/dx

$\therefore \frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

using equation 2 and 3, we get –

$\Rightarrow \frac{dy}{dx} = \frac{(1-3^x)(3^x\log 3)-(1+3^x)(-3^x\log 3)}{(1-3^x)^2}$

$\Rightarrow \frac{dy}{dx} = \frac{3^x\log 3\,(1-3^x+1+3^x)}{(1-3^x)^2}$

$\Rightarrow \frac{dy}{dx} = \frac{2\times 3^x\log 3}{(1-3^x)^2}$

Hence,

$$\frac{dy}{dx} = \frac{2 \times 3^x\log 3}{(1-3^x)^2} \text{ ....ans}$$

19. Question

Differentiate the following functions with respect to x:

$$\frac{3^x}{x + \tan x}$$

Answer

Let, $y = \frac{3^x}{x + \tan x}$

We have to find dy/dx

As we can observe that y is a fraction of two functions say u and v where,

$u = 3^x$ and $v = x + \tan x$

∴ y = u/v

As we know that to find the derivative of fraction of two function we apply quotient rule of differentiation.

By quotient rule, we have –

$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$ ...equation 1

As, $u = 3^x$

$\because \frac{d}{dx}(a^x) = a^x \log a$, so we get –

$\therefore \frac{du}{dx} = \frac{d}{dx}(3^x) = 3^x \log_e 3$ ...equation 2

As, v = x + tan x

$\because \frac{d}{dx}(\tan x) = \sec^2 x$, so we get –

$\therefore \frac{dv}{dx} = \frac{d}{dx}(x + \tan x) = 1 + \sec^2 x$ ...equation 3

∴ from equation 1, we can find dy/dx

$\therefore \frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

using equation 2 and 3, we get –

$\Rightarrow \frac{dy}{dx} = \frac{(x+\tan x)(3^x \log 3)-(3^x)(1+\sec^2 x)}{(x+\tan x)^2}$

$\Rightarrow \frac{dy}{dx} = \frac{3^x (x\log 3 + \tan x \log 3 - \sec^2 x - 1)}{(x+\tan x)^2}$

Hence,

$$\frac{dy}{dx} = \frac{3^x (x\log 3 + \tan x \log 3 - \sec^2 x - 1)}{(x+\tan x)^2} \text{ .... ans}$$

20. Question

Differentiate the following functions with respect to x:

$$\frac{1+\log x}{1-\log x}$$

Answer

Let, y = $\frac{1+\log x}{1-\log x}$

We have to find dy/dx

As we can observe that y is a fraction of two functions say u and v where,

u = 1 + log x and v = 1 – log x

∴ y = u/v

As we know that to find the derivative of fraction of two function we apply quotient rule of differentiation.

By quotient rule, we have –

$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$ ...equation 1

As, u = 1 + log x

$\because \frac{d}{dx}(\log x) = \frac{1}{x}$, so we get –

$\therefore \frac{du}{dx} = \frac{d}{dx}(1 + \log x) = 0 + \frac{1}{x} = \frac{1}{x}$ ...equation 2

As, v = 1 – log x

$\because \frac{d}{dx}(\log x) = \frac{1}{x}$, so we get –

$\therefore \frac{dv}{dx} = \frac{d}{dx}(1 - \log x) = -\frac{1}{x}$ ...equation 3

∴ from equation 1, we can find dy/dx

$\therefore \frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

using equation 2 and 3, we get –

$\Rightarrow \frac{dy}{dx} = \frac{(1-\log x)\left(\frac{1}{x}\right)-(1+\log x)\left(-\frac{1}{x}\right)}{(1-\log x)^2}$

$\Rightarrow \frac{dy}{dx} = \frac{\frac{1}{x}(1-\log x + 1 + \log x)}{(1-\log x)^2}$

$\Rightarrow \frac{dy}{dx} = \frac{2}{x(1-\log x)^2}$

Hence,

$$\frac{dy}{dx} = \frac{2}{x(1-\log x)^2} \text{ .... ans}$$

21. Question

Differentiate the following functions with respect to x:

$$\frac{4x + 5\sin x}{3x + 7\cos x}$$

Answer

Let, y = $\frac{4x+5\sin x}{3x+7\cos x}$

We have to find dy/dx

As we can observe that y is a fraction of two functions say u and v where,

u = 4x + 5sin x and v = 3x + 7cos x

∴ y = u/v

As we know that to find the derivative of fraction of two function we apply quotient rule of differentiation.

By quotient rule, we have –

$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$ ...equation 1

As, u = 4x + 5 sin x

∵ $\frac{d}{dx}(\sin x) = \cos x$, so we get –

∴ $\frac{du}{dx} = \frac{d}{dx}(4x + 5\sin x) = 4 + 5\cos x$ ...equation 2

As, v = 3x + 7 cos x

∵ $\frac{d}{dx}(\cos x) = -\sin x$, so we get –

∴ $\frac{dv}{dx} = \frac{d}{dx}(3x + 7\cos x) = 3 - 7\sin x$ ...equation 3

∴ from equation 1, we can find dy/dx

∴ $\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

using equation 2 and 3, we get –

$\Rightarrow \frac{dy}{dx} = \frac{(3x+7\cos x)(4+5\cos x)-(4x+5\sin x)(3-7\sin x)}{(3x+7\cos x)^2}$

$\Rightarrow \frac{dy}{dx} = \frac{12x+15x\cos x+28\cos x+35\cos^2 x-12x+28x\sin x-15\sin x+35\sin^2 x}{(3x+7\cos x)^2}$

$\Rightarrow \frac{dy}{dx} = \frac{15x\cos x+28\cos x+35\cos^2 x+35\sin^2 x+28x\sin x-15\sin x}{(3x+7\cos x)^2}$

$\Rightarrow \frac{dy}{dx} = \frac{15x\cos x+28\cos x+35+28x\sin x-15\sin x}{(3x+7\cos x)^2}$

Hence,

$$\frac{dy}{dx} = \frac{15x\cos x + 28\cos x + 28x\sin x - 15\sin x + 35}{(3x + 7\cos x)^2} \text{ .... ans}$$

22. Question

Differentiate the following functions with respect to x:

$$\frac{x}{1+\tan x}$$

Answer

Let, $y = \frac{x}{1+\tan x}$

We have to find dy/dx

As we can observe that y is a fraction of two functions say u and v where,

u = x and v = 1 + tan x

∴ y = u/v

As we know that to find the derivative of fraction of two function we apply quotient rule of differentiation.

By quotient rule, we have –

$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$ ...equation 1

As, u = x

∴ $\frac{du}{dx} = 1$ ...equation 2 {∵ $\frac{d}{dx}(x^n) = nx^{n-1}$ }

As, v = 1 + tan x

∵ $\frac{d}{dx}(\tan x) = \sec^2 x$, so we get –

∴ $\frac{dv}{dx} = \frac{d}{dx}(1 + \tan x) = 0 + \sec^2 x = \sec^2 x$ ...equation 3

∴ from equation 1, we can find dy/dx

∴ $\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

⇒ $\frac{dy}{dx} = \frac{(1+\tan x)(1)-(x)(\sec^2 x)}{(1+\tan x)^2}$ {using equation 2 and 3}

⇒ $\frac{dy}{dx} = \frac{1+\tan x - x\sec^2 x}{(1+\tan x)^2}$

Hence,

$$\frac{dy}{dx} = \frac{1 + \tan x - x\sec^2 x}{(1 + \tan x)^2} \text{ ....ans}$$

23. Question

Differentiate the following functions with respect to x:

$$\frac{a + b\sin x}{c + d\cos x}$$

Answer

Let, $y = \frac{a + b\sin x}{c + d\cos x}$

We have to find dy/dx

As we can observe that y is a fraction of two functions say u and v where,

u = a + bsin x and v = c + d cos x

∴ y = u/v

As we know that to find the derivative of fraction of two function we apply quotient rule of differentiation.

By quotient rule, we have –

$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$ ...equation 1

As, u = a + b sinx

$\because \frac{d}{dx}(\sin x) = \cos x$, so we get –

$\therefore \frac{du}{dx} = \frac{d}{dx}(a + b\sin x) = 0 + b\cos x = b\cos x$ ...equation 2

As, v = c + d cos x

$\because \frac{d}{dx}(\cos x) = -\sin x$, so we get –

$\therefore \frac{dv}{dx} = \frac{d}{dx}(c + d\cos x) = 0 - d\sin x = -d\sin x$ ...equation 3

∴ from equation 1, we can find dy/dx

$\therefore \frac{dy}{dx} = \frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$

using equation 2 and 3, we get –

$\Rightarrow \frac{dy}{dx} = \frac{(c + d\cos x)(b\cos x)-(a + b\sin x)(-d\sin x)}{(c + d\cos x)^2}$

$\Rightarrow \frac{dy}{dx} = \frac{(bc\cos x + bd\cos^2 x + ad\sin x + bd\sin^2 x)}{(c + d\cos x)^2}$

$\because \sin^2 x + \cos^2 x = 1$, so we get –

$\Rightarrow \frac{dy}{dx} = \frac{(bc\cos x + ad\sin x + bd(\cos^2 x + \sin^2 x)}{(c + d\cos x)^2} = \frac{bc\cos x + ad\sin x + bd}{(c + d\cos x)^2}$

$\Rightarrow \frac{dy}{dx} = \frac{bc\cos x + ad\sin x + bd}{(c + d\cos x)^2}$

Hence,

$$\frac{dy}{dx} = \frac{bc\cos x + ad\sin x + bd}{(c + d\cos x)^2} \text{ .... ans}$$

24. Question

Differentiate the following functions with respect to x:

$$\frac{px^2 + qx + r}{ax + b}$$

Answer

Let, $y = \frac{px^2 + qx + r}{ax + b}$

We have to find dy/dx

As we can observe that y is a fraction of two functions say u and v where,

u = px^2 + qx + r and v = ax + b

∴ y = u/v

As we know that to find the derivative of fraction of two function we apply quotient rule of differentiation.

By quotient rule, we have –

$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$ ...equation 1

As, u = px^2 + qx + r

$\therefore \frac{du}{dx} = 2px + q$ ...equation 2 {$\because \frac{d}{dx}(x^n) = nx^{n-1}$ }

As, v = ax + b

$\because \frac{d}{dx}(x^n) = nx^{n-1}$, so we get –

$\therefore \frac{dv}{dx} = \frac{d}{dx}(ax + b) = a$ ...equation 3

∴ from equation 1, we can find dy/dx

$\therefore \frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

$\Rightarrow \frac{dy}{dx} = \frac{(ax+b)(2px+q)-(px^2+qx+r)(a)}{(ax+b)^2}$ {using equation 2 and 3}

$\Rightarrow \frac{dy}{dx} = \frac{2apx^2+aqx+2bpx+bq-apx^2-aqx-ar}{(ax+b)^2}$

$\Rightarrow \frac{dy}{dx} = \frac{apx^2+2bpx+bq-ar}{(ax+b)^2}$

Hence,

$$\frac{dy}{dx} = \frac{apx^2 + 2bpx + bq - ar}{(ax + b)^2} \text{ .... ans}$$

25. Question

Differentiate the following functions with respect to x:

$$\frac{x^n}{\sin x}$$

Answer

Let, y = $\frac{x^n}{\sin x}$

We have to find dy/dx

As we can observe that y is a fraction of two functions say u and v where,

u = x^n and v = sin x

∴ y = u/v

As we know that to find the derivative of fraction of two function we apply quotient rule of differentiation.

By quotient rule, we have –

$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$ ...equation 1

As, u = x^n

$\because \frac{d}{dx}(x^n) = nx^{n-1}$, so we get –

$\therefore \frac{du}{dx} = \frac{d}{dx}(x^n) = nx^{n-1}$ ...equation 2

As, v = sin x

$\because \frac{d}{dx}(\sin x) = \cos x$, so we get –

$\therefore \frac{dv}{dx} = \frac{d}{dx}(\sin x) = \cos x$ ...equation 3

∴ from equation 1, we can find dy/dx

$\therefore \frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

using equation 2 and 3, we get –

$\Rightarrow \frac{dy}{dx} = \frac{\sin x(nx^{n-1}) - (x^n)(\cos x)}{\sin^2 x}$

Hence,

$$\frac{dy}{dx} = \frac{\sin x\,(nx^{n-1}) - (x^n)(\cos x)}{\sin^2 x} \text{ .... ans}$$

26. Question

Differentiate the following functions with respect to x:

$$\frac{x^5 - \cos x}{\sin x}$$

Answer

Let, $y = \frac{x^5 - \cos x}{\sin x}$

We have to find dy/dx

As we can observe that y is a fraction of two functions say u and v where,

u = x^5 – cos x and v = sin x

∴ y = u/v

As we know that to find the derivative of fraction of two function we apply quotient rule of differentiation.

By quotient rule, we have –

$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$ ...equation 1

As, u = x^5 – cos x

$\because \frac{d}{dx}(x^n) = nx^{n-1}$ & $\frac{d}{dx}(\cos x) = -\sin x$, so we get –

$\therefore \frac{du}{dx} = \frac{d}{dx}(x^5 - \cos x) = 5x^4 + \sin x$ ...equation 2

As, v = sin x

$\because \frac{d}{dx}(\sin x) = \cos x$, so we get –

$\therefore \frac{dv}{dx} = \frac{d}{dx}(\sin x) = \cos x$ ...equation 3

∴ from equation 1, we can find dy/dx

$\therefore \frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

using equation 2 and 3, we get –

$$\Rightarrow \frac{dy}{dx} = \frac{\sin x\left(5x^4 + \sin x\right) - \left(x^5 - \cos x\right)(\cos x)}{\sin^2 x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{5x^4 \sin x - x^5 \cos x + (\sin^2 x + \cos^2 x)}{\sin^2 x}$$

$\because \sin^2 x + \cos^2 x = 1$, so we get –

$$\Rightarrow \frac{dy}{dx} = \frac{5x^4 \sin x - x^5 \cos x + 1}{\sin^2 x}$$

Hence,

$$\frac{dy}{dx} = \frac{x^4(5\sin x - x\cos x)\ +\ 1}{\sin^2 x} \text{ .... ans}$$

27. Question

Differentiate the following functions with respect to x:

$$\frac{x + \cos x}{\tan x}$$

Answer

Let, $y = \frac{x + \cos x}{\tan x}$

We have to find dy/dx

As we can observe that y is a fraction of two functions say u and v where,

u = x + cos x and v = tan x

∴ y = u/v

As we know that to find the derivative of fraction of two function we apply quotient rule of differentiation.

By quotient rule, we have –

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2} \text{ ...equation 1}$$

As, u = x + cos x

$\because \frac{d}{dx}(x^n) = nx^{n-1}$ & $\frac{d}{dx}(\cos x) = -\sin x$, so we get –

$$\therefore \frac{du}{dx} = \frac{d}{dx}(x + \cos x) = 1 - \sin x \text{ ...equation 2}$$

As, v = tan x

$\because \frac{d}{dx}(\tan x) = \sec^2 x$, so we get –

$$\therefore \frac{dv}{dx} = \frac{d}{dx}(\tan x) = \sec^2 x \text{ ...equation 3}$$

∴ from equation 1, we can find dy/dx

$$\therefore \frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

using equation 2 and 3, we get –

$$\Rightarrow \frac{dy}{dx} = \frac{(\tan x)(1 - \sin x) - (x + \cos x)(\sec^2 x)}{\tan^2 x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{\tan x - \sin x \tan x - x\sec^2 x - x\sec x}{\sin^2 x}$$

Hence,

$$\frac{dy}{dx} = \frac{\tan x - \sin x \tan x - x \sec^2 x - x \sec x}{\sin^2 x} \text{ .... ans}$$

28. Question

Differentiate the following functions with respect to x:

$$\frac{x^n}{\sin x}$$

Answer

Let, $y = \frac{x^n}{\sin x}$

We have to find dy/dx

As we can observe that y is a fraction of two functions say u and v where,

$u = x^n$ and v = sin x

∴ y = u/v

As we know that to find the derivative of fraction of two function we apply quotient rule of differentiation.

By quotient rule, we have –

$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$ ...equation 1

As, $u = x^n$

$\because \frac{d}{dx}(x^n) = nx^{n-1}$, so we get –

$\therefore \frac{du}{dx} = \frac{d}{dx}(x^n) = nx^{n-1}$ ...equation 2

As, v = sin x

$\because \frac{d}{dx}(\sin x) = \cos x$, so we get –

$\therefore \frac{dv}{dx} = \frac{d}{dx}(\sin x) = \cos x$ ...equation 3

∴ from equation 1, we can find dy/dx

$\therefore \frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

using equation 2 and 3, we get –

$\Rightarrow \frac{dy}{dx} = \frac{\sin x\left(nx^{n-1}\right) - (x^n)(\cos x)}{\sin^2 x}$

Hence,

$$\frac{dy}{dx} = \frac{\sin x\,(nx^{n-1}) - (x^n)(\cos x)}{\sin^2 x} \text{ .... ans}$$

29. Question

Differentiate the following functions with respect to x:

$$\frac{ax + b}{px^2 + qx + r}$$

Answer

Let, $y = \frac{ax + b}{px^2 + qx + r}$

We have to find dy/dx

As we can observe that y is a fraction of two functions say u and v where,

u = ax + b and v = px^2 + qx + r

∴ y = u/v

As we know that to find the derivative of fraction of two function we apply quotient rule of differentiation.

By quotient rule, we have –

$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$ ...equation 1

As, u = ax + b

∴ $\frac{du}{dx} = a$ ...equation 2 {∵ $\frac{d}{dx}(x^n) = nx^{n-1}$ }

As, v = px^2 + qx + r

∵ $\frac{d}{dx}(x^n) = nx^{n-1}$, so we get –

∴ $\frac{dv}{dx} = \frac{d}{dx}(px^2 + qx + r) = 2px + q$ ...equation 3

∴ from equation 1, we can find dy/dx

∴ $\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

⇒ $\frac{dy}{dx} = \frac{(px^2 + qx + r)(a) - (ax + b)(2px + q)}{(px^2 + qx + r)^2}$ {using equation 2 and 3}

⇒ $\frac{dy}{dx} = \frac{(apx^2 + aqx + ar) - 2apx^2 - aqx - 2bpx - bq}{(px^2 + qx + r)^2}$

⇒ $\frac{dy}{dx} = \frac{-apx^2 - 2bpx + ar - bq}{(px^2 + qx + r)^2}$

Hence,

$$\frac{dy}{dx} = \frac{-apx^2 - 2bpx + ar - bq}{(px^2 + qx + r)^2} \text{ ....ans}$$

30. Question

Differentiate the following functions with respect to x:

$$\frac{1}{ax^2 + bx + c}$$

Answer

Let, $y = \frac{1}{ax^2 + bx + c}$

We have to find dy/dx

As we can observe that y is a fraction of two functions say u and v where,

u = 1 and v = ax^2 + bx + c

∴ y = u/v

As we know that to find the derivative of fraction of two function we apply quotient rule of differentiation.

By quotient rule, we have –

$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$ ...equation 1

As, u = 1

$\therefore \frac{du}{dx} = 0$ ...equation 2 {$\because \frac{d}{dx}(x^n) = nx^{n-1}$ }

As, v = ax^2 + bx + c

$\because \frac{d}{dx}(x^n) = nx^{n-1}$, so we get –

$\therefore \frac{dv}{dx} = \frac{d}{dx}(ax^2 + bx + c) = 2ax + b$ ...equation 3

∴ from equation 1, we can find dy/dx

$\therefore \frac{dy}{dx} = \frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$

$\Rightarrow \frac{dy}{dx} = \frac{(ax^2+bx+c)(0)-(1)(2ax+b)}{(ax^2+bx+c)^2}$ {using equation 2 and 3}

$\Rightarrow \frac{dy}{dx} = \frac{-(2ax+b)}{(ax^2+bx+c)^2}$

Hence,

$$\frac{dy}{dx} = -\frac{(2ax + b)}{(ax^2 + bx + c)^2} \ldots.\text{ans}$$

Very Short Answer

1. Question

Write the value of $\lim\limits_{x\to c}\frac{f(x)-f(c)}{x-c}$

Answer

By definition of derivative we know that derivative of a function at a given real number say c is given by:

$f'(c) = \lim\limits_{x\to c}\frac{f(x)-f(c)}{x-c}$

∴ as per the definition of derivative of a function at a given real number we can say that –

$\lim\limits_{x\to c}\frac{f(x)-f(c)}{x-c} = f'(c)$

2. Question

Write the value of $\lim\limits_{x\to a}\frac{xf(a)-af(x)}{x-a}$

Answer

By definition of derivative we know that derivative of a function at a given real number say c is given by :

$f'(c) = \lim\limits_{x\to c}\frac{f(x)-f(c)}{x-c}$

let Z = $\lim\limits_{x\to a}\frac{xf(a)-af(x)}{x-a}$

If somehow we got a form similar to that of in definition of derivative we can write it in a simpler form.

$\therefore Z = \lim_{x \to a} \frac{(x-a)f(a) - af(x) + af(a)}{x-a}$ {adding & subtracting af(a) in numerator}

$\Rightarrow Z = \lim_{x \to a} \frac{(x-a)f(a)}{x-a} - a \lim_{x \to a} \frac{f(x)-f(a)}{x-a}$ {using algebra of limits}

Using the definition of the derivative , we have –

$\Rightarrow Z = \lim_{x \to a} f(a) - af'(a)$

$\therefore Z = f(a) - a f'(a)$

3. Question

If x < 2, then write the value of $\frac{d}{dx}\left(\sqrt{x^2 - 4x + 4}\right)$

Answer

Let $y = \sqrt{x^2 - 4x + 4}$

Now,

$$\frac{dy}{dx} = \frac{1}{\sqrt{x^2 - 4x + 4}} \times \frac{d}{dx}(x^2 - 4x + 4)$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{x^2 - 4x + 4}} \times (2x - 4)$$

From above,

$x^2 - 4x + 4 > 0$

$(x - 2)^2 > 0$

$x > 2$

But x < 2. Therefore, $\frac{dy}{dx}$ does not exist for the given function.

4. Question

If $\frac{\pi}{2} < x < \pi$, then find $\frac{d}{dx}\left(\sqrt{\frac{1 + \cos 2x}{2}}\right)$

Answer

As we know that $1+\cos 2x = 2\sin^2 x$

As, $\pi/2 < x < \pi$

$\therefore$ sin x will be positive.

Let $Z = \frac{d}{dx}\left(\sqrt{\frac{1+\cos 2x}{2}}\right)$

$\Rightarrow Z = \frac{d}{dx}\left(\sqrt{\frac{2\sin^2 x}{2}}\right)$

$\Rightarrow Z = \frac{d}{dx}(|\sin x|)$ { as we need to consider positive square root}

$\because$ sin x is positive

$\therefore Z = \frac{d}{dx}(\sin x)$

We know that differentiation of sin x is cos x

Hence,

Z = cos x

5. Question

Write the value of $\frac{d}{dx}(x\,|\,x\,|)$

Answer

As we need to differentiate f(x) = x |x|

We know the property of mod function that

$$|x| = \begin{cases} -x, x < 0 \\ x, x \geq 0 \end{cases}$$

$$\therefore f(x) = x\,|x| = \begin{cases} -x^2, x < 0 \\ x^2, x \geq 0 \end{cases}$$

Hence,

$$\frac{d}{dx}(f(x)) = \begin{cases} \frac{d}{dx}(-x^2), x < 0 \\ \frac{d}{dx}(x^2), x \geq 0 \end{cases}$$

As $\frac{d}{dx}(x^n) = nx^{n-1}$

$$\therefore \frac{d}{dx}(x|x|) = \begin{cases} -2x, x < 0 \\ 2x, x \geq 0 \end{cases}$$

6. Question

Write the value of $\frac{d}{dx}\{(x+|\,x\,|)\,|\,x\,|\}$

Answer

As we need to differentiate f(x) = (x+|x|)|x|

We know the property of a mod function that

$$|x| = \begin{cases} -x, x < 0 \\ x, x \geq 0 \end{cases}$$

$$\therefore f(x) = (x + |x|)|x| = \begin{cases} (x + (-x))(-x) = 0, x < 0 \\ (x + x)x = 2x^2, x \geq 0 \end{cases}$$

Hence,

$$\frac{d}{dx}(f(x)) = \begin{cases} \frac{d}{dx}(0), x < 0 \\ \frac{d}{dx}(2x^2), x \geq 0 \end{cases}$$

As $\frac{d}{dx}(x^n) = nx^{n-1}$

$$\therefore \frac{d}{dx}\{(x + |x|)|x|\} = \begin{cases} 0, x < 0 \\ 4x, x \geq 0 \end{cases}$$

7. Question

If f(x) = |x| + |x – 1|, write the value of $\frac{d}{dx}(f(x))$.

Answer

As we need to differentiate f(x) = |x| + |x – 1|

We know the property of mod function that–

$$|x| = \begin{cases} -x, x < 0 \\ x, x \geq 0 \end{cases}$$

$$\therefore f(x) = |x| + |x-1| = \begin{cases} x + x - 1 = 2x - 1, x \geq 1 \\ x + \{-(x-1)\} = 1, 0 < x < 1 \\ -x - (x-1) = -2x + 1, x \leq 0 \end{cases}$$

$$\therefore f(x) = \begin{cases} 2x - 1, x \geq 1 \\ 1, 0 < x < 1 \\ -2x + 1, x \leq 0 \end{cases}$$

Hence,

$$\frac{d}{dx}(f(x)) = \begin{cases} \frac{d}{dx}(2x-1), x \geq 1 \\ \frac{d}{dx}(1), 0 < x < 1 \\ \frac{d}{dx}(-2x+1), x \leq 0 \end{cases}$$

As $\frac{d}{dx}(x^n) = nx^{n-1}$

$$\therefore \frac{d}{dx}\{f(x)\} = \begin{cases} 2, x \geq 1 \\ 0, 0 < x < 1 \\ -2, x \leq 0 \end{cases}$$

8. Question

Write the value of the derivation of f(x) = |x – 1| + |x – 3| at x = 2.

Answer

As we need to differentiate f(x) = |x – 3| + |x – 1|

We know the property of a mod function that–

$$|x| = \begin{cases} -x, x < 0 \\ x, x \geq 0 \end{cases}$$

$$\therefore f(x) = |x-3| + |x-1| = \begin{cases} x - 3 + x - 1 = 2x - 4, x \geq 3 \\ -x + 3 + \{(x-1)\} = 2, 1 < x < 3 \\ -x + 3 - (x-1) = -2x + 4, x \leq 1 \end{cases}$$

$$\therefore f(x) = \begin{cases} 2x - 4, x \geq 3 \\ 2, 1 < x < 3 \\ -2x + 4, x \leq 1 \end{cases}$$

Hence,

$$\frac{d}{dx}(f(x)) = \begin{cases} \frac{d}{dx}(2x-4), x \geq 3 \\ \frac{d}{dx}(2), 1 < x < 3 \\ \frac{d}{dx}(-2x+4), x \leq 1 \end{cases}$$

As $\frac{d}{dx}(x^n) = nx^{n-1}$

$$\therefore \frac{d}{dx}\{f(x)\} = \begin{cases} 2, x \geq 3 \\ 0, 1 < x < 3 \\ -2, x \leq 1 \end{cases}$$

From above equation, we can say that

value of derivative at x = 2 is 0 ⇒ f'(2) = 0

9. Question

If $f(x)=\frac{x^2}{|x|}$, write $\frac{d}{dx}(f(x))$

Answer

As we need to differentiate $f(x)=\frac{x^2}{|x|}$

We know the property of mod function that

$$|x|=\begin{cases}-x, x<0\\ x, x\geq 0\end{cases}$$

$$\therefore f(x)=\frac{x^2}{|x|}=\begin{cases}\frac{x^2}{-x}=-x, x<0\\ \frac{x^2}{x}=x, x>0\end{cases}$$

Hence,

$$\frac{d}{dx}(f(x))=\begin{cases}\frac{d}{dx}(-x), x<0\\ \frac{d}{dx}(x), x>0\end{cases}$$

As $\frac{d}{dx}(x^n)=nx^{n-1}$

$$\therefore \frac{d}{dx}\{f(x)\}=\begin{cases}-1, x<0\\ 1, x>0\end{cases}$$

Note: f(x) is not differentiable at x = 0 because left hand derivative of f(x) is not equal to right hand derivative at x = 0

10. Question

Write the value of $\frac{d}{dx}(\log|x|)$

Answer

As we need to differentiate f(x) = log |x|

We know the property of a mod function that

$$|x|=\begin{cases}-x, x<0\\ x, x\geq 0\end{cases}$$

$$\therefore f(x)=\log|x|=\begin{cases}\log(-x), x<0\\ \log x, x>0\end{cases}$$

Note: log x is not defined at x = 0. So its derivative at x = 0 also does not exist.

Hence,

$$\frac{d}{dx}(f(x))=\begin{cases}\frac{d}{dx}(\log(-x)), x<0\\ \frac{d}{dx}(\log x), x>0\end{cases}$$

As $\frac{d}{dx}(\log x)=\frac{1}{x}$

$\therefore \frac{d}{dx}\{f(x)\} = \begin{cases} -\frac{1}{x}, x < 0 \\ \frac{1}{x}, x > 0 \end{cases}$

11. Question

If f(1) = 1, f'(1) = 2, then write the value of $\lim_{x\to1}\frac{\sqrt{f(x)}-1}{\sqrt{x}-1}$

Answer

By definition of derivative we know that derivative of a function at a given real number say c is given by :

$f'(c) = \lim_{x\to c}\frac{f(x)-f(c)}{x-c}$

let $Z = \lim_{x\to1}\frac{\sqrt{f(x)}-1}{\sqrt{x}-1}$

As Z is taking 0/0 form because f(1) = 1

So on rationalizing the Z, we have–

$Z = \lim_{x\to1}\frac{\sqrt{f(x)}-1}{\sqrt{x}-1}\times\frac{\sqrt{f(x)}+1}{\sqrt{f(x)}+1}\times\frac{\sqrt{x}+1}{\sqrt{x}+1}$

$\Rightarrow Z = \lim_{x\to1}\frac{\left\{\left(\sqrt{f(x)}\right)^2-1^2\right\}\left(\sqrt{x}+1\right)}{\left(\sqrt{f(x)}+1\right)\left(\left(\sqrt{x}\right)^2-1^2\right)}$ {using $a^2–b^2$ = (a+b)(a–b)}

$\Rightarrow Z = \lim_{x\to1}\frac{\{f(x)-1\}\left(\sqrt{x}+1\right)}{\left(\sqrt{f(x)}+1\right)(x-1)}$

Using algebra of limits, we have –

$Z = \lim_{x\to1}\frac{\{f(x)-1\}}{(x-1)}\times\lim_{x\to1}\frac{\left(\sqrt{x}+1\right)}{\left(\sqrt{f(x)}+1\right)}$

Using the definition of the derivative, we have –

$Z = f'(1)\times\frac{\left(\sqrt{1}+1\right)}{\left(\sqrt{f(1)}+1\right)}$

⇒ Z = 2 × (2/2) = 2 {using values given in equation}

∴ Z = 2

12. Question

Write the derivation of f(x) = 3|2 + x| at x = –3.

Answer

As we need to differentiate f(x) = 3|2+x|

We know the property of a mod function that

$|x| = \begin{cases} -x, x < 0 \\ x, x \geq 0 \end{cases}$

$\therefore f(x) = 3|2+x| = \begin{cases} 3(2-x) = 6-3x, x < -2 \\ (2+x)3 = 6+3x, x \geq -2 \end{cases}$

Hence,

$\frac{d}{dx}(f(x)) = \begin{cases} \frac{d}{dx}(6-3x), x < -2 \\ \frac{d}{dx}(6+3x), x \geq -2 \end{cases}$

As $\frac{d}{dx}(x^n) = nx^{n-1}$

$\therefore \frac{d}{dx}\{f(x)\} = \begin{cases} -3, x < -2 \\ 3, x \geq -2 \end{cases}$

Clearly form the above equation we can say that,

Value of derivative at x = –3 is –3 i.e. f’(–3) = –3

13. Question

If |x| < 1 and y = 1 + x + x^2 + x^3 + ..., then write the value of $\frac{dy}{dx}$.

Answer

As |x| < 1

And y = 1+x+x^2 +....... (this is an infinite G.P with common ratio x)

∵ |x|<1

∴ using the formula for sum of infinite G.P,we have–

$y = \frac{1}{1-x}$

We know that f’(ax+b) = af’(x)

As $\frac{d}{dx}\left(\frac{1}{x}\right) = -\frac{1}{x^2}$

But here a = –1 and b = 1

$\therefore \frac{dy}{dx} = \frac{d}{dx}\left(\frac{1}{1-x}\right) = \frac{1}{(1-x)^2}$

14. Question

If $f(x) = \log_{x^2} x^3$, write the value of f’(x).

Answer

Given,

$f(x) = \log_{x^2} x^3$

Applying change of base formula, we have –

$f(x) = \frac{\log_e x^3}{\log_e x^2} = \frac{3\log_e x}{2\log_e x} = \frac{3}{2}$ {using properties of log}

As differentiation of constant term is 0

∴ f’(x) = 0

MCQ

1. Question

Let f(x) = x – [x], x ∈ R, then $f'\left(\frac{1}{2}\right)$ is

A. $\frac{3}{2}$

B. 1

C. 0

D. –1

Answer

As we need to differentiate f(x) = x – [x]

We know the property of greatest integer function that

$F'(x) = 1 - \frac{d}{dx}[x]$

F'(x) = 1 – 0 = 1

∴ option (b) is correct answer.

2. Question

If $f(x) = \frac{x-4}{2\sqrt{x}}$, then f'(1) is

A. $\frac{5}{4}$

B. $\frac{4}{5}$

C. 1

D. 0

Answer

As, f(x) = $\frac{x-4}{2\sqrt{x}} = \frac{1}{2}\left\{\frac{x}{\sqrt{x}} - \frac{4}{\sqrt{x}}\right\} = \frac{1}{2}\left\{x^{\frac{1}{2}} - 4x^{-\frac{1}{2}}\right\}$

We know that,

$\frac{d}{dx}(x^n) = nx^{n-1}$

∴ f'(x) = $\frac{d}{dx}\big(f(x)\big) = \frac{1}{2}\frac{d}{dx}(x^{\frac{1}{2}} - 4x^{-\frac{1}{2}})$

⇒ f'(x) = $\frac{1}{2}\left\{\frac{d}{dx}\left(x^{\frac{1}{2}}\right) - 4\frac{d}{dx}\left(x^{-\frac{1}{2}}\right)\right\} = \frac{1}{2}\left\{\frac{1}{2}x^{-\frac{1}{2}} - 4\left(-\frac{1}{2}\right)x^{-\frac{1}{2}-1}\right\}$

⇒ f'(x) = $\frac{1}{2}\left\{\frac{1}{2}x^{-\frac{1}{2}} + 2x^{-\frac{3}{2}}\right\}$

∴ f'(1) = $\frac{1}{2}\left\{\frac{1}{2}(1)^{-\frac{1}{2}} + 2(1)^{-\frac{3}{2}}\right\} = \frac{1}{2}\left\{\frac{1}{2} + 2\right\} = \frac{5}{4}$

∴ f'(1) = 5/4

Clearly from above calculation only 1 answer is possible which is 5/4

∴ option (a) is the only correct answer.

3. Question

If $y = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots..$, then $\frac{dy}{dx}$

A. y + 1

B. y – 1

C. y

D. y^2

Answer

As, $y = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$

We know that,

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\therefore \frac{dy}{dx} = \frac{d}{dx}\left(1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots\right)$$

$$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}(1) + \frac{d}{dx}\left(\frac{x}{1!}\right) + \frac{d}{dx}\left(\frac{x^2}{2!}\right) + \cdots$$

$$\Rightarrow \frac{dy}{dx} = 0 + 1 + \frac{2x}{2!} + \frac{3x^2}{3!} + \cdots$$

$$\Rightarrow \frac{dy}{dx} = 1 + \frac{x}{1} + \frac{x^2}{2!} + \cdots$$

Clearly, in comparison with y, we can say that–

$$\frac{dy}{dx} = y$$

Hence,

Option (c) is the only correct answer.

4. Question

If f(x) = 1 – x + x^2 – x^3 + ….. – x^{99} + x^{100}, then f'(1) equals

A. 150

B. –50

C. –150

D. 50

Answer

As, f(x) = 1 – x + x^2 – x^3 + ….. – x^{99} + x^{100}

We know that,

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\therefore \frac{dy}{dx} = \frac{d}{dx}(1 - x + x^2 - x^3 + \cdots - x^{99} + x^{100})$$

$$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}(1) - \frac{d}{dx}(x) + \frac{d}{dx}(x^2) - \frac{d}{dx}(x^3) \ldots - \frac{d}{dx}(x^{99}) + \frac{d}{dx}(x^{100})$$

$$\Rightarrow \frac{dy}{dx} = 0 - 1 + 2x - 3x^2 + \cdots - 99x^{98} + 100x^{99}$$

⇒ f'(1) = –1+2–3+…–99+100

⇒ f'(1) = (2+4+6+8+….+98+100) – (1+3+5+…+97+99)

Both terms have 50 terms

We know that sum of n terms of an A.P = $\frac{n}{2}(a_1 + a_n)$

∴ f'(1) = $\frac{50}{2}(2 + 100) - \frac{50}{2}(1 + 99) = 25(102 - 100) = 50$

Clearly above solution suggests that only 1 result is possible which is 50.

Hence,

Only option (d) is the correct answer.

5. Question

If $y = \frac{1+\frac{1}{x^2}}{1-\frac{1}{x^2}}$, then $\frac{dy}{dx} =$

A. $-\frac{4x}{(x^2-1)^2}$

B. $-\frac{4x}{x^2-1}$

C. $\frac{1-x^2}{4x}$

D. $\frac{4x}{x^2-1}$

Answer

As $y = \frac{1+\frac{1}{x^2}}{1-\frac{1}{x^2}} = \frac{x^2+1}{x^2-1}$

To calculate dy/dx, we can use the quotient rule.

From quotient rule we know that : $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$

$\therefore \frac{dy}{dx} = \frac{d}{dx}\left(\frac{x^2+1}{x^2-1}\right) = \frac{(x^2-1)\frac{d}{dx}(x^2+1)-(x^2+1)\frac{d}{dx}(x^2-1)}{(x^2-1)^2}$

We know that, $\frac{d}{dx}(x^n) = nx^{n-1}$

$\Rightarrow \frac{dy}{dx} = \frac{(x^2-1)(2x)-(x^2+1)(2x)}{(x^2-1)^2} = \frac{2x(x^2-1-x^2-1)}{(x^2-1)^2}$

$\Rightarrow \frac{dy}{dx} = \frac{-4x}{(x^2-1)^2}$

Clearly, from above solution we can say that option (a) is the only correct answer.

6. Question

If $y = \sqrt{x} + \frac{1}{\sqrt{x}}$, then $\frac{dy}{dx}$ at $x = 1$ is

A. 1

B. $\frac{1}{2}$

C. $\frac{1}{\sqrt{2}}$

D. 0

Answer

As, $y = \sqrt{x} + \frac{1}{\sqrt{x}}$

⇒ y = $x^{1/2} + x^{-1/2}$

We know that: $\frac{d}{dx}(x^n) = nx^{n-1}$

$\therefore \frac{dy}{dx} = \frac{d}{dx}\left(x^{\frac{1}{2}}\right) + \frac{d}{dx}\left(x^{-\frac{1}{2}}\right)$

$\Rightarrow \frac{dy}{dx} = \frac{1}{2}x^{\frac{1}{2}-1} - \frac{1}{2}x^{-\frac{1}{2}-1} = \frac{1}{2}\left(x^{-\frac{1}{2}} - x^{-\frac{3}{2}}\right)$

$\therefore \left(\frac{dy}{dx}\right)_{\text{at } x=1} = \frac{1}{2}\left(1^{-\frac{1}{2}} - 1^{-\frac{3}{2}}\right) = 0$

Clearly, only option (d) matches with our result.

∴ option (d) is the only correct choice.

7. Question

If f(x) = $x^{100} + x^{99}$ + ...+ x + 1, then f'(1) is equal to

A. 5050

B. 5049

C. 5051

D. 50051

Answer

As, f(x) = 1 + x + $x^2 + x^3$ + ... + $x^{99} + x^{100}$

We know that,

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$\therefore \frac{dy}{dx} = \frac{d}{dx}(1 + x + x^2 + x^3 + \cdots + x^{99} + x^{100})$

$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}(1) + \frac{d}{dx}(x) + \frac{d}{dx}(x^2) + \frac{d}{dx}(x^3) \ldots + \frac{d}{dx}(x^{99}) + \frac{d}{dx}(x^{100})$

$\Rightarrow \frac{dy}{dx} = 0 + 1 + 2x + 3x^2 + \cdots + 99x^{98} + 100x^{99}$

⇒ f'(1) = 1 + 2 + 3 + ... + 99 + 100 (total 100 terms)

We know that sum of n terms of an A.P $= \frac{n}{2}(a_1 + a_n)$

∴ f'(1) $= \frac{100}{2}(1 + 100) = 50(100 + 1) = 50 \times 101 = 5050$

Clearly, the above solution suggests that only 1 result is possible which is 5050.

Hence,

The only option (a) is the correct answer.

8. Question

If $f(x) = 1 + x + \frac{x^2}{2} + \ldots + \frac{x^{100}}{100}$, then f'(1) is equal to

A. $\frac{1}{100}$

B. 100

C. 50

D. 0

Answer

As, $y = 1 + x + \frac{x^2}{2} + \frac{x^3}{3} + ... + \frac{x^{100}}{100}$

We know that,

$\frac{d}{dx}(x^n) = nx^{n-1}$

$\therefore \frac{dy}{dx} = f'(x) = \frac{d}{dx}(1 + x + \frac{x^2}{2} + \frac{x^3}{3} + \cdots + \frac{x^{100}}{100})$

$\Rightarrow f'(x) = \frac{d}{dx}(1) + \frac{d}{dx}(x) + \frac{d}{dx}\left(\frac{x^2}{2}\right) + \cdots + \frac{d}{dx}\left(\frac{x^{100}}{100}\right)$

$\Rightarrow f'(x) = 0 + 1 + \frac{2x}{2} + \frac{3x^2}{3} + \cdots + \frac{100x^{99}}{100}$

$\Rightarrow f'(x) = 1 + x + x^2 + \cdots + x^{99}$

⇒ f'(1) = 1+1+1+......1 (100 terms) = 100

Clearly above solution suggests that only 1 result is possible which is 100.

Hence,

Only option (b) is the correct answer.

9. Question

If $y = \frac{\sin x + \cos x}{\sin x - \cos x}$, then $\frac{dy}{dx}$ at $x = 0$ is

A. -2

B. 0

C. 1/2

D. does not exist

Answer

As $y = \frac{\sin x + \cos x}{\sin x - \cos x}$

To calculate dy/dx, we can use the quotient rule.

From quotient rule we know that : $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

$\therefore \frac{dy}{dx} = \frac{d}{dx}\left(\frac{\sin x + \cos x}{\sin x - \cos x}\right) = \frac{(\sin x - \cos x)\frac{d}{dx}(\sin x + \cos x) - (\sin x + \cos x)\frac{d}{dx}(\sin x - \cos x)}{(\sin x - \cos x)^2}$

We know that, $\frac{d}{dx}(\sin x) = \cos x$ & $\frac{d}{dx}(\cos x) = -\sin x$

$\Rightarrow \frac{dy}{dx} = \frac{(\sin x - \cos x)(\cos x - \sin x) - (\sin x + \cos x)(\cos x + \sin x)}{(\sin x - \cos x)^2}$

$\Rightarrow \frac{dy}{dx} = \frac{-(\sin x - \cos x)^2 - (\sin x + \cos x)^2}{(\sin x - \cos x)^2}$

$\therefore \left(\frac{dy}{dx}\right)_{at\ x=0} = \frac{-(\sin 0 - \cos 0)^2 - (\sin 0 + \cos 0)^2}{(\sin 0 - \cos 0)^2} = \frac{-1-1}{1} = -2$

$\Rightarrow \frac{dy}{dx} = -2$

10. Question

If $y = \frac{\sin(x+9)}{\cos x}$, then $\frac{dy}{dx}$ at x = 0 is

A. cos 9

B. sin 9

C. 0

D. 1

Answer

$y = \frac{\sin(x+9)}{\cos x}$

From quotient rule we know that : $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

Differentiating, we get,

$\frac{dy}{dx} = \frac{\cos x \cos(x+9) + \sin x \sin(x+9)}{(\cos x)^2}$

$\left(\frac{dy}{dx}\right)$ at $(x = 0) = \cos 9$

Hence, a is the answer.

11. Question

If $f(x) = \frac{x^n - a^n}{x - a}$, then f′(a) is

A. 1

B. 0

C. $\frac{1}{2}$

D. does not exist

Answer

As, $f(x) = \frac{x^n - a^n}{x - a}$

To calculate dy/dx we can use quotient rule.

From quotient rule we know that : $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

$\therefore f'(x) = \frac{d}{dx}\left(\frac{x^n - a^n}{x - a}\right) = \frac{(x-a)\frac{d}{dx}(x^n - a^n) - (x^n - a^n)\frac{d}{dx}(x-a)}{(x-a)^2}$

We know that, $\frac{d}{dx}(x^n) = nx^{n-1}$

$\Rightarrow f'(x) = \frac{(x-a)(nx^{n-1})-(x^n-a^n)(1)}{(x-a)^2}$

∵ x – a is a factor of $x^n – a^n$, we can write:

$x^n – a^n = (x–a)(x^{n-1}+ax^{n-2}+a^2x^{n-3}+\ldots+a^{n-1})$

$\Rightarrow f'(x) = \frac{(x-a)(nx^{n-1})-(x-a)(x^{n-1}+ax^{n-2}+\cdots+a^{n-1})(1)}{(x-a)^2}$

$\Rightarrow f'(x) = \frac{(x-a)\{(nx^{n-1})-(x^{n-1}+ax^{n-2}+\cdots+a^{n-1})\}}{(x-a)^2}$

$\Rightarrow f'(x) = \frac{\{(nx^{n-1})-(x^{n-1}+ax^{n-2}+\cdots+a^{n-1})\}}{x-a}$

$\therefore f'(a) = \frac{\{(na^{n-1})-(a^{n-1}+a\times a^{n-2}+\cdots+a^{n-1})\}}{a-a}$

$\Rightarrow f'(a) = \frac{\{(na^{n-1})-(a^{n-1}+a\times a^{n-2}+\cdots+a^{n-1})\}}{0} = \text{does not exist}$

Clearly form above solution we can say that option (d) is the only correct answer.

12. Question

If f(x) = x sin x, then $f'(\pi/2) =$

A. 0

B. 1

C. –1

D. $\frac{1}{2}$

Answer

As, f(x) = x sin x

To calculate dy/dx we can use product rule.

From quotient rule we know that : $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

$\therefore f'(x) = \frac{d}{dx}(x\sin x) = x\frac{d}{dx}(\sin x) + \sin x\frac{d}{dx}(x)$

We know that, $\frac{d}{dx}(x^n) = nx^{n-1}$ & $\frac{d}{dx}(\sin x) = \cos x$

$\Rightarrow f'(x) = x\cos x + \sin x$

$\therefore f'\left(\frac{\pi}{2}\right) = \left(\frac{\pi}{2}\right)\cos\frac{\pi}{2} + \sin\frac{\pi}{2} = 0 + 1 = 1$

Clearly form above solution we can say that option (b) is the only correct answer as the solution has only1 possible answer which matches with option (b) only.